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ON THE MULTIDIMENSIONAL PREDICTION AND FILTERING PROBLEM AND THE FACTORIZATION OF SPECTRAL MATRICES

BY

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ABSTRACT

Since the linear prediction and filtering of multiple correlated stationary random signals involves the solution of a matrix integral equation, several methods of solution of this equation have been developed. In this paper the direct solution by complex variable techniques is extended from the one-dimensional case. The problem reduces to the factorization of a spectral density matrix and this factorization is the principal topic of the paper.

Under very general conditions, a factorization procedure has been developed by Wiener and Masani (1)[3] for discrete time series. Their technique is applied here with some modifications to the factorization of the spectral density matrices of continuous processes. A simplified procedure is developed also for the case where the elements of the spectral density matrix are rational functions of frequency. Examples illustrate the general technique and the simplified procedure.

INTRODUCTION

In the theory of the linear prediction and filtering of correlated stationary random signals, the form of the desired multidimensional network is specified by the solution of a matrix integral equation (2)

$$\int_0^\infty \mathbf{R}(\tau - \sigma)\tilde{\mathbf{h}}(\sigma)d\sigma = \mathbf{R}_m(\tau), \qquad 0 \le \tau \le \infty, \tag{1}$$

where $\mathbf{R}(\tau - \sigma)$ is a matrix containing as elements the covariance functions of the received waveforms $e_j(t)$, that is,

$$R_{jk}(\tau) = E\{e_j(t)e_k(t + \tau)\}, \tag{2}$$

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Further, if $m_j(t)$ are the desired outputs, then the outputs of the optimum network are given by

$$m_j{}^*(t) = \sum_k \int_0^\infty h_{jk}(\tau)e_k(t - \tau)d\tau, \tag{3}$$

and the elements of $\mathbf{R}_m(\tau)$ by

$$R_{m\,jk}(\tau) = E\{e_j(t)m_k(t + \tau)\}. \tag{4}$$

Thus the matrices occurring in Eq. 1 are defined by Eqs. 2, 3, and 4, with the notation $\sim$ denoting transpose in Eq. 1.

The method of undetermined coefficients was proposed originally by Wiener (2) for the solution of Eq. 1. An alternate method of solution (3) is to reduce Eq. 1 to a system of simultaneous differential equations with constant coefficients, and represents a generalization of the work of Zadeh and Ragazzini (4). It is the purpose of this paper to call attention to a third technique which may not be widely known.

A MATRIX WIENER-HOPF APPROACH

In a manner analogous to the one-dimensional case (2, 5) Eq. 1 can be rewritten as

$$\int_0^\infty \mathbf{R}(\tau - \sigma)\tilde{\mathbf{h}}(\sigma)d\sigma - \mathbf{R}_m(\tau) = \mathbf{f}(\tau), \qquad -\infty \leq \tau \leq \infty, \tag{5}$$

with

$$\mathbf{f}(\tau) = 0, \qquad \tau > 0. \tag{6}$$

The Fourier transforms of $\mathbf{R}(\tau)$, $\mathbf{h}(\tau)$, $\mathbf{R}_m(\tau)$ and $\mathbf{f}(\tau)$ may be denoted by $\mathbf{\Phi}(i\omega)$, $\mathbf{H}(i\omega)$, $\mathbf{\Phi}_m(i\omega)$ and $\mathbf{F}(i\omega)$, respectively, for example,

$$\mathbf{\Phi}(i\omega) \triangleq \int_{-\infty}^\infty e^{-i\omega\tau}\mathbf{R}(\tau)d\tau.$$

Taking transforms of both sides of Eq. 5 results in

$$\mathbf{\Phi}(i\omega)\tilde{\mathbf{H}}(i\omega) - \mathbf{\Phi}_m(i\omega) = \mathbf{F}(i\omega). \tag{7}$$

It follows from Eq. 6 that $\mathbf{F}(i\omega)$ is analytic and bounded in the upper half ω-plane (6).

Now, let $\mathbf{\Phi}(i\omega)$ be factored into the form

$$\mathbf{\Phi}(i\omega) = \mathbf{\Psi}^\dagger(i\omega)\mathbf{\Psi}(i\omega), \tag{8}$$

where

$$\mathbf{\Psi}^\dagger(i\omega) \triangleq \tilde{\mathbf{\Psi}}(-i\omega). \tag{9}$$

The factorization given by Eq. 8 is to be such that the matrix $\mathbf{\Psi}(i\omega)$ is element-by-element analytic and bounded in the lower half ω-plane, and its determinant Det $\mathbf{\Psi}(i\omega)$ is free of zeros in the lower half ω-plane. These two properties of $\mathbf{\Psi}(i\omega)$ imply that $\mathbf{\Psi}(i\omega)$ and its inverse $\mathbf{\Psi}^{-1}(i\omega)$ both represent transfer functions of physically realizable multidimensional networks.[4] The problem of realizing the factorization of Eq. 8 will be deferred until the next section.

Multiplying Eq. 7 from the left by $[\mathbf{\Psi}^{\dagger}(i\omega)]^{-1}$ yields

$$\mathbf{\Psi}(i\omega)\tilde{\mathbf{H}}(i\omega) - [\mathbf{\Psi}^{\dagger}(i\omega)]^{-1}\mathbf{\Phi}_m(i\omega) = [\mathbf{\Psi}^{\dagger}(i\omega)]^{-1}\mathbf{F}(i\omega). \tag{10}$$

Now, since both $\mathbf{\Psi}(i\omega)$ and $\tilde{\mathbf{H}}(i\omega)$ are analytic and bounded in the lower half ω-plane, the time response of $\mathbf{\Psi}(i\omega)\tilde{\mathbf{H}}(i\omega)$ must vanish for negative time, that is,

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}[\mathbf{\Psi}(i\omega)\tilde{\mathbf{H}}(i\omega)]e^{i\omega\tau}d\omega = 0, \qquad \tau < 0. \tag{11}$$

Similarly, it follows that

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\{[\mathbf{\Psi}^{\dagger}(i\omega)]^{-1}\,\mathbf{F}(i\omega)\}e^{i\omega\tau}d\omega = 0, \qquad \tau > 0. \tag{12}$$

Therefore, the time response of $\mathbf{\Psi}(i\omega)\tilde{\mathbf{H}}(i\omega)$ must be equal to the non-negative-time portion of the time response of $[\mathbf{\Psi}^{\dagger}(i\omega)]^{-1}\mathbf{\Phi}_m(i\omega)$. This fact can be expressed by the relationship

$$\mathbf{\Psi}(i\omega)\tilde{\mathbf{H}}(i\omega) = \frac{1}{2\pi}\int_{0}^{\infty}e^{-i\omega\tau}\left\{\int_{-\infty}^{\infty}e^{iu\tau}[\mathbf{\Psi}^{\dagger}(iu)]^{-1}\mathbf{\Phi}_m(iu)du\right\}d\tau. \tag{13}$$

From Eq. 13, the transposed transfer matrix $\tilde{\mathbf{H}}(i\omega)$ is given by

$$\tilde{\mathbf{H}}(i\omega) = \mathbf{\Psi}^{-1}(i\omega)\int_{0}^{\infty}e^{-i\omega\tau}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{iu\tau}[\mathbf{\Psi}^{\dagger}(iu)]^{-1}\mathbf{\Phi}_m(iu)du\right\}d\tau, \tag{14}$$

which represents a generalization of the one-dimensional solution (5).

THE FACTORIZATION OF THE SPECTRAL DENSITY MATRIX

It can be seen that an explicit solution of Eq. 1 can be found provided that the factorization (Eq. 8) of the spectral density matrix $\mathbf{\Phi}(i\omega)$ can be effected. One factorization technique has been developed recently by Youla (7). Youla's technique is applicable whenever $\mathbf{\Phi}(i\omega)$ is element-by-element a rational function of ω. An earlier factorization procedure was developed by Wiener and Masani (1) for discrete time series. Their technique is applied here with some modifications to the factorization of the spectral density matrices of continuous processes. The procedure will now be explained and illustrated.

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[4] Thus $\mathbf{\Psi}(i\omega)$ may be said to be the transfer function of a minimum-phase multi-dimensional network.

The matrix $\mathbf{\Phi}(i\omega)$ to be factored is assumed to satisfy the following conditions:

(1) $\mathbf{\Phi}(i\omega)$ is Hermitian on the real ω-axis, that is,

$$\mathbf{\Phi}(i\omega) = \mathbf{\Phi}^{\dagger}(i\omega) \tag{15}$$

for ω real.

(2) The determinant Det $\mathbf{\Phi}(i\omega)$ satisfies the inequality

$$\int_{-\infty}^{\infty} \frac{|\ln|\text{Det } \mathbf{\Phi}(i\omega)||}{k^2 + \omega^2} d\omega < \infty, \tag{16}$$

where $| \ |$ denotes the absolute value, and k is any real finite positive constant (see Appendix).

(3) Let the smallest and largest eigenvalues of $\mathbf{\Phi}(i\omega)$ on the real ω-axis be λ_1 and λ_2, respectively, then

$$0 < \lambda_1 < \lambda_2 < \infty. \tag{17}$$

Conditions (1) and (2) are satisfied for all physical processes. Condition (3) is satisfied if the spectral density matrix $\mathbf{\Phi}(i\omega)$ is determined experimentally, as shown by Wiener and Masani (1), but need not be satisfied by theoretically postulated spectral density matrices.

We note that the diagonal terms $\Phi_{jj}(i\omega)$ of the matrix $\mathbf{\Phi}(i\omega)$ represent auto-spectral densities, and can be factored easily into the form

$$\Phi_{jj}(\omega) = \Psi_{jj}(-i\omega)\Psi_{jj}(i\omega), \tag{18}$$

where the $\Psi_{jj}(i\omega)$ are analytic, bounded and free of zeros in the lower half ω-plane. Now let $\mathbf{\Gamma}(i\omega)$ be a diagonal matrix with elements

$$\Gamma_{jk}(i\omega) \triangleq \frac{1}{\Psi_{jj}(i\omega)} \delta_{jk}. \tag{19}$$

It is apparent that the matrix $\mathbf{\Phi}'(i\omega)$, defined by

$$\mathbf{\Phi}'(i\omega) = \mathbf{\Gamma}^{\dagger}(i\omega)\mathbf{\Phi}(i\omega)\mathbf{\Gamma}(i\omega), \tag{20}$$

has diagonal terms which are each equal to unity. Let the matrix $\mathbf{M}(i\omega)$ be obtained by subtracting a unit matrix $\mathbf{1}$ from $\mathbf{\Phi}'(i\omega)$; that is,

$$\mathbf{M}(i\omega) = \mathbf{\Phi}'(i\omega) - \mathbf{1}. \tag{21}$$

The matrix $\mathbf{M}(i\omega)$ can be written in the form

$$\mathbf{M}(i\omega) = \mathbf{M}_0 + \mathbf{M}_+(i\omega) + \mathbf{M}_-(i\omega), \tag{22}$$

where $\mathbf{M}_0$ is a constant matrix, $\mathbf{M}_+(i\omega)$ is element-by-element analytic and bounded in the lower half ω-plane and $\mathbf{M}_-(i\omega)$ is element-by-element analytic and bounded in the upper half ω-plane. It is recognized that these requirements do not uniquely determine the decomposition (18). However, it is shown in the Appendix that different decompositions satisfying the analytic requirements yield equivalent results. In practice it is frequently convenient to add the requirement that

$$\mathbf{M}_+(\infty) = \mathbf{M}_-(\infty) = 0, \tag{23}$$

which makes the decomposition (18) unique. Now let the matrix $\mathbf{N}(i\omega)$ be defined by the series

$$\mathbf{N}(i\omega) \triangleq \mathbf{1} - \mathbf{M}_+ + (\mathbf{M}\mathbf{M}_+)_+ - [\mathbf{M}(\mathbf{M}\mathbf{M}_+)_+]_+ + \cdots, \tag{24}$$

where the subscript $+$ has the same meaning as in Eq. 22. It follows immediately from the work of Wiener and Masani (1) that

(1) The matrices $\mathbf{N}(i\omega)$ and $\mathbf{N}^{-1}(i\omega)$ are element-by-element analytic and bounded in the lower half ω-plane.

(2) The matrix $\mathbf{G}$, defined by

$$\mathbf{G} \triangleq \mathbf{N}^{\dagger}(i\omega)\boldsymbol{\Phi}'(i\omega)\mathbf{N}(i\omega), \tag{25}$$

is a constant symmetric matrix.

Furthermore, it is seen from Eq. 24 that

$$\mathbf{N}^{\dagger}(\infty) = \mathbf{N}(\infty) = \mathbf{1}, \tag{26}$$

if Eq. 23 applies. Therefore,

$$\mathbf{G} = \boldsymbol{\Phi}'(\infty) \tag{27}$$

since $\mathbf{G}$ is a constant matrix. Thus $\mathbf{G}$ can be found without performing the matrix multiplications implied by Eq. 25.

From Eqs. 20 and 25, $\boldsymbol{\Phi}(i\omega)$ can be written as

$$\boldsymbol{\Phi}(i\omega) = [\boldsymbol{\Gamma}^{\dagger}(i\omega)]^{-1}[\mathbf{N}^{\dagger}(i\omega)]^{-1}\mathbf{G}\mathbf{N}^{-1}(i\omega)\boldsymbol{\Gamma}^{-1}(i\omega). \tag{28}$$

All that remains to complete the factorization is to factor $\mathbf{G}$. This factorization is easily accomplished since $\mathbf{G}$ is a constant matrix. For example, $\mathbf{G}$ may be diagonalized by unitary operations and the square root taken of the resultant diagonal matrix. However, there is no need to proceed further than Eq. 28. Using arguments similar to those leading to Eq. 14, it is easy to show that, instead of Eq. 14, the solution can be expressed as

$$\tilde{\mathbf{H}}(i\omega) = \mathbf{\Gamma}(i\omega)\mathbf{N}(i\omega)\mathbf{G}^{-1}\int_0^\infty d\tau e^{-i\omega\tau} \times \left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iu\tau}\mathbf{N}^\dagger(iu)\mathbf{\Gamma}^\dagger(iu)\mathbf{\Phi}_m(iu)du\right\}. \quad (29)$$

A SIMPLIFIED PROCEDURE FOR RATIONAL SPECTRA

Since Eq. 24 is an infinite series, $\mathbf{N}(i\omega)$ cannot always be obtained in closed form. If $\mathbf{\Phi}'(i\omega)$ is element-by-element a rational function of $i\omega$, an alternate procedure yields $\mathbf{N}(i\omega)$ in a finite number of steps.

To derive this procedure, $\mathbf{\Phi}'(i\omega)$ is written as

$$\mathbf{\Phi}'(i\omega) = \mathbf{1} + \mathbf{M}_0 + \mathbf{M}_+(i\omega) + \mathbf{M}_-(i\omega). \quad (30)$$

It should be noted that the successive terms in the series (24) for $\mathbf{N}(i\omega)$ can be written more explicitly; for example,

$$(\mathbf{M}\mathbf{M}_+)_+ = \mathbf{M}_0\mathbf{M}_+ + \mathbf{M}_+\mathbf{M}_+ + (\mathbf{M}_-\mathbf{M}_+)_+. \quad (31)$$

If the product $\mathbf{\Phi}'(i\omega)\mathbf{N}(i\omega)$ is formed and use is made of Eqs. 30 and 31, it is found that most terms cancel and the product becomes

$$\mathbf{\Phi}'(i\omega)\mathbf{N}(i\omega) = \mathbf{1} + \mathbf{M}_0 + [\mathbf{M}_-(i\omega)\mathbf{N}(i\omega)]_-. \quad (32)$$

From Eq. 32, $\mathbf{N}(i\omega)$ can be written formally as

$$\mathbf{N}(i\omega) = [\mathbf{\Phi}'(i\omega)]^{-1}\{\mathbf{1} + \mathbf{M}_0 + [\mathbf{M}_-(i\omega)\mathbf{N}(i\omega)]_-\}. \quad (33)$$

Since $\mathbf{N}(i\omega)$ is analytic and bounded in the lower half ω-plane and $[\mathbf{M}_-(i\omega)\mathbf{N}(i\omega)]_-$ is analytic and bounded in the upper half ω-plane, the poles of $\mathbf{N}(i\omega)$ must result from the upper half ω-plane poles of $[\mathbf{\Phi}'(i\omega)]^{-1}$.

Let $[\mathbf{\Phi}'(i\omega)]^{-1}$ and $\mathbf{M}_-(i\omega)$ be written as

$$[\mathbf{\Phi}'(i\omega)]^{-1} = [\mathbf{\Phi}'(\infty)]^{-1} + \sum_{m=1}^{K}\mathbf{A}_m(\sigma_m + i\omega)^{-1} + \sum_{m=1}^{K}\tilde{\mathbf{A}}_m(\sigma_m - i\omega)^{-1}, \quad (34)$$

and

$$\mathbf{M}_-(i\omega) = \sum_{n=1}^{L}\mathbf{B}_n(\tau_n - i\omega)^{-1}. \quad (35)$$

From Eq. 34 it is apparent that $\mathbf{N}(i\omega)$ can be written as

$$\mathbf{N}(i\omega) = \mathbf{1} + \sum_{m=1}^{K} \mathbf{C}_m(\sigma_m + i\omega)^{-1}. \tag{36}$$

Substituting Eqs. 34 through 36 into Eq. 33 and equating coefficients of the upper half ω-plane poles yields

$$\begin{aligned}\mathbf{C}_m &= \mathbf{A}_m(\mathbf{1} + \mathbf{M}_0) + \mathbf{A}_m \sum_{n=1}^{L} \mathbf{B}_n(\sigma_m + \tau_n)^{-1} \\ &\quad + \mathbf{A}_m \sum_{n=1}^{L} \sum_{k=1}^{K} \mathbf{B}_n\mathbf{C}_k(\sigma_k + \tau_n)^{-1}(\sigma_m + \tau_n)^{-1}, \quad m = 1, 2, \cdots, K.\end{aligned} \tag{37}$$

Equation 37 represents a set of K matrix equations from which the K unknown constant matrices $\mathbf{C}_m$ can be found.

Example I

A two-dimensional prediction problem is considered in some detail. Let two correlated signals $e_1(\tau)$ and $e_2(\tau)$ be received in the interval $-\infty \leq \tau \leq t$. It is desired to estimate $e_1(t + \alpha)$ and $e_2(t + \alpha)$ for $\alpha > 0$. The spectral density matrix of $e_1(\tau)$ and $e_2(\tau)$ is given as[5]

$$\mathbf{\Phi}(i\omega) = \begin{bmatrix} \dfrac{1}{1 + \omega^2} & \dfrac{\epsilon}{(1 - i\omega)^2} \\[2ex] \dfrac{\epsilon}{(1 + i\omega)^2} & \dfrac{1}{1 + \omega^2} \end{bmatrix}, \qquad |\epsilon| < 1. \tag{38}$$

The matrix $\mathbf{\Phi}_m(i\omega)$ for this case is simply

$$\mathbf{\Phi}_m(i\omega) = e^{i\omega\alpha}\mathbf{\Phi}(i\omega). \tag{39}$$

From Eq. 20 it is found that

$$\mathbf{\Phi}'(i\omega) = \begin{bmatrix} 1 & \epsilon\dfrac{1 + i\omega}{1 - i\omega} \\[2ex] \epsilon\dfrac{1 - i\omega}{1 + i\omega} & 1 \end{bmatrix}. \tag{40}$$

In this example the power spectral density matrix is a rational function of $i\omega$; hence the simplified procedure previously described can be applied. For comparison the problem is solved using first the general approach and then this simplified procedure.

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[5] This example appeared in (2) and was solved by the method of undetermined coefficients.

The unit matrix **1** may be subtracted from Eq. 40 to yield

$$\mathbf{M}(i\omega) = \begin{bmatrix} 0 & \epsilon\dfrac{1+i\omega}{1-i\omega} \\ \epsilon\dfrac{1-i\omega}{1+i\omega} & 0 \end{bmatrix}, \tag{41}$$

and, finally,

$$\mathbf{M}_+(i\omega) = \begin{bmatrix} 0 & 0 \\ \dfrac{2\epsilon}{1+i\omega} & 0 \end{bmatrix}. \tag{42}$$

For this simple problem the series (24) for $\mathbf{N}(\omega)$ terminates after two terms, and the result is

$$\mathbf{N}(i\omega) = \begin{bmatrix} 1 & 0 \\ \dfrac{-2\epsilon}{1+i\omega} & 1 \end{bmatrix}. \tag{43}$$

From Eq. 25 the matrix **G** is given by

$$\mathbf{G} = \begin{bmatrix} 1 & \dfrac{-2\epsilon}{1-i\omega} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \epsilon\dfrac{1+i\omega}{1-i\omega} \\ \epsilon\dfrac{1-i\omega}{1+i\omega} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{-2\epsilon}{1+i\omega} & 1 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 1 & -\epsilon \\ -\epsilon & 1 \end{bmatrix}. \tag{44}$$

It is seen that **G** is indeed a constant matrix. From Eq. 29 and 39 the solution is found to be

$$\tilde{\mathbf{H}}(i\omega) = \mathbf{\Gamma}(i\omega)\mathbf{N}(i\omega)\int_0^\infty d\tau e^{-i\omega\tau}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iu\tau}\mathbf{N}^{-1}(iu)\mathbf{\Gamma}^{-1}(iu)du\right] \tag{45}$$

$$= e^{-\alpha}\begin{bmatrix} 1 & 0 \\ 2\epsilon\alpha & 1 \end{bmatrix}.$$

Following the simplified procedure of the previous section, Eqs. 30 and 40 are used to obtain

$$\mathbf{\Phi}'(\infty) = \mathbf{1} + \mathbf{M}_0 = \begin{bmatrix} 1 & -\epsilon \\ -\epsilon & 1 \end{bmatrix}. \tag{46}$$

The matrix given by Eq. 40 may be inverted to yield

$$[\mathbf{\Phi}'(i\omega)]^{-1} = \frac{1}{1-\epsilon^2}\begin{bmatrix} 1 & -\epsilon\dfrac{1+i\omega}{1-i\omega} \\ -\epsilon\dfrac{1-i\omega}{1+i\omega} & 1 \end{bmatrix}. \tag{47}$$

A comparison of Eqs. 34 and 47 shows that $\sigma_1 = 1$ and that

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 \\ \dfrac{-2\epsilon}{1-\epsilon^2} & 0 \end{bmatrix}. \tag{48}$$

From Eq. 41, $\mathbf{M}_-(i\omega)$ is given by

$$\mathbf{M}_-(i\omega) = \begin{bmatrix} 0 & \dfrac{2\epsilon}{1-i\omega} \\ 0 & 0 \end{bmatrix}, \tag{49}$$

and Eq. 35 yields $\tau_1 = 1$ and

$$\mathbf{B}_1 = \begin{bmatrix} 0 & 2\epsilon \\ 0 & 0 \end{bmatrix}. \tag{50}$$

Substitution of Eqs. 46, 48, and 50 into Eq. 37 yields a matrix equation which may be solved for $\mathbf{C}_1$ to give

$$\mathbf{C}_1 = \begin{bmatrix} 0 & 0 \\ -2\epsilon & 0 \end{bmatrix}. \tag{51}$$

From Eq. 36, $\mathbf{N}(i\omega)$ may be written as

$$\mathbf{N}(i\omega) = \mathbf{1} + \begin{bmatrix} 0 & 0 \\ -2\epsilon & 0 \end{bmatrix}\frac{1}{1+i\omega}, \tag{52}$$

which is identical to Eq. 43 obtained by the general procedure.

Example II

In this example a somewhat more complicated spectral density matrix will be factored. Let this matrix $\mathbf{\Phi}(i\omega)$ be

$$\mathbf{\Phi}(i\omega) = \begin{bmatrix} \dfrac{\alpha^2}{\alpha^2 + \omega^2} & \dfrac{\epsilon\alpha\beta}{(\alpha + i\omega)(\beta - i\omega)} \\[2ex] \dfrac{\epsilon\alpha\beta}{(\alpha - i\omega)(\beta + i\omega)} & \dfrac{\beta^2}{\beta^2 + \omega^2} \end{bmatrix}, \qquad |\epsilon| < 1. \tag{53}$$

The matrix $\mathbf{\Gamma}(i\omega)$ of Eq. 19 is found, by factoring the diagonal terms, to be

$$\mathbf{\Gamma}(i\omega) = \begin{bmatrix} \dfrac{\alpha + i\omega}{\alpha} & 0 \\[2ex] 0 & \dfrac{\beta + i\omega}{\beta} \end{bmatrix}. \tag{54}$$

From Eqs. 20 and 54, $\mathbf{\Phi}'(i\omega)$ is found to be

$$\mathbf{\Phi}'(i\omega) = \begin{bmatrix} 1 & \epsilon\dfrac{\alpha - i\omega}{\alpha + i\omega}\dfrac{\beta + i\omega}{\beta - i\omega} \\[2ex] \epsilon\dfrac{\alpha + i\omega}{\alpha - i\omega}\dfrac{\beta - i\omega}{\beta + i\omega} & 1 \end{bmatrix}. \tag{55}$$

Thus $\mathbf{M}(i\omega)$ of Eq. 21 is given by

$$\mathbf{M}(i\omega) = \begin{bmatrix} 0 & \epsilon\dfrac{\alpha - i\omega}{\alpha + i\omega}\dfrac{\beta + i\omega}{\beta - i\omega} \\[2ex] \epsilon\dfrac{\alpha + i\omega}{\alpha - i\omega}\dfrac{\beta - i\omega}{\beta + i\omega} & 0 \end{bmatrix}. \tag{56}$$

From Eqs. 22, 23, and 56 the matrix $\mathbf{M}_+(i\omega)$ is

$$\mathbf{M}_+(i\omega) = \begin{bmatrix} 0 & \epsilon\dfrac{2\alpha}{\alpha + i\omega}\dfrac{\beta - \alpha}{\beta + \alpha} \\[2ex] \epsilon\dfrac{2\beta}{\beta + i\omega}\dfrac{\alpha - \beta}{\alpha + \beta} & 0 \end{bmatrix}. \tag{57}$$

The series (24) for $\mathbf{N}(i\omega)$ in this example is a geometric series, and can be evaluated to yield

$$\mathbf{N}(i\omega) = \begin{bmatrix} 1 + \dfrac{k_1}{\alpha + i\omega} & k_2 \dfrac{\alpha}{\alpha + i\omega} \\ -k_2 \dfrac{\beta}{\beta + i\omega} & 1 - \dfrac{k_1}{\beta + i\omega} \end{bmatrix}, \tag{58}$$

where the constants k_1 and k_2 are given by

$$k_1 = \frac{4\epsilon^2\alpha\beta(\alpha - \beta)}{(\alpha + \beta)^2 - 4\epsilon^2\alpha\beta}, \tag{59}$$

and

$$k_2 = \frac{2\epsilon(\alpha^2 - \beta^2)}{(\alpha + \beta)^2 - 4\epsilon^2\alpha\beta}. \tag{60}$$

Application of Eq. 27 yields immediately

$$\mathbf{G} = \begin{bmatrix} 1 & \epsilon \\ \epsilon & 1 \end{bmatrix}. \tag{61}$$

The matrix **G** could be obtained also from Eq. 25. Since this example has a rational spectral density, it could be solved by the simplified procedure previously discussed.

APPENDIX

In order that continuous processes may be brought within the framework of the work of Wiener and Masani (1), a mapping of the ω-plane into some z-plane must be effected such that the real axis of the ω-plane is transformed into the unit circle in the z-plane, the upper half ω-plane into the region outside the unit circle in the z-plane and the lower half ω-plane into the region inside the unit circle. Such a mapping can be obtained by the transformation

$$z = \frac{ik + \omega}{ik - \omega}, \qquad 0 < k < \infty. \tag{62}$$

Physically, the constant k can be interpreted as a scale factor which translates the dimensionless time scale of the discrete case into the time scale of the continuous case. It will be shown that the transformation (62) also serves to give a precise definition to the decomposition (22).

Let $F(z)$ be a function which can be represented by the expansion

$$F(z) = \sum_{n=-\infty}^{\infty} a_n z^n. \tag{63}$$

Then the functions $F_+(z)$, $F_-(z)$, and F_0 are defined as

$$F_0 \triangleq a_0 \tag{64}$$

$$F_+(z) \triangleq \sum_{n=1}^{\infty} a_n z^n \tag{65}$$

$$F_-(z) \triangleq \sum_{n=1}^{\infty} a_{-n} z^{-n}. \tag{66}$$

It is apparent from Eqs. 65 and 66 that

$$F_+(0) = F_-(\infty) = 0. \tag{67}$$

From Eq. 62 it is seen that the points $z = 0$, ∞ correspond to $\omega = -ik$, ik, respectively. Therefore, the condition to be imposed on the decomposition (22) is

$$\mathbf{M}_+(-ik) = \mathbf{M}_-(ik) = 0, \qquad 0 < k < \infty. \tag{68}$$

It may appear from Eq. 68 that the condition (23) is inapplicable, since k is restricted to be finite. This, however, is not the case. Consider the alternative decompositions of $\mathbf{M}(i\omega)$

$$\mathbf{M}(i\omega) = \mathbf{M}_+(i\omega) + \mathbf{M}_-(i\omega) + \mathbf{M}_0 = \mathbf{M}^+(i\omega) + \mathbf{M}^-(i\omega) + \mathbf{M}^0, \tag{69}$$

where $\mathbf{M}_+$ and $\mathbf{M}^+$ are analytic in the lower half ω-plane, $\mathbf{M}_-$ and $\mathbf{M}^-$ analytic in the upper half ω-plane and $\mathbf{M}_0$ and $\mathbf{M}^0$ are constants. Furthermore, require that

$$\mathbf{M}_+(-ik) = \mathbf{M}_-(ik) = 0, \qquad 0 < k < \infty, \tag{70}$$

and

$$\mathbf{M}^+(\infty) = \mathbf{M}^-(\infty) = 0. \tag{71}$$

It is apparent that $\mathbf{M}^+$ and $\mathbf{M}_+$, thus defined, differ only by an additive matrix constant. Now let $\mathbf{N}(i\omega)$ be given by

$$\mathbf{N}(i\omega) = 1 - \mathbf{M}_+ + (\mathbf{M}\mathbf{M}_+)_+ - [\mathbf{M}(\mathbf{M}\mathbf{M}_+)_+]_+ + \cdots, \tag{72}$$

and let $\mathbf{N}'(i\omega)$ be given by

$$\mathbf{N}'(i\omega) = 1 - \mathbf{M}^+ + (\mathbf{M}\mathbf{M}^+)^+ - [\mathbf{M}(\mathbf{M}\mathbf{M}^+)^+]^+ + \cdots. \tag{73}$$

Equation 73 can be rewritten as

$$\begin{aligned}\mathbf{N}'(i\omega) &= 1 - (\mathbf{M}_+ + \mathbf{K}_1) + \{[\mathbf{M}(\mathbf{M}_+ + \mathbf{K}_1)]_+ + \mathbf{K}_2\} \cdots \\ &= \{1 - \mathbf{M}_+ + (\mathbf{M}\mathbf{M}_+)_+ - [\mathbf{M}(\mathbf{M}\mathbf{M}_+)_+]_+ + \cdots\}\{1 - \mathbf{K}_1 + \mathbf{K}_2 - \mathbf{K}_3 + \cdots\} \\ &= \mathbf{N}(i\omega)\mathbf{K}.\end{aligned} \tag{74}$$

Thus the matrices $\mathbf{N}'(i\omega)$ and $\mathbf{N}(i\omega)$ differ only by a multiplicative constant matrix. This difference does not affect the final result given by Eq. 29. To see that such is the case, instead of Eq. 29, write

$$\tilde{\mathbf{H}}'(i\omega) = \mathbf{\Gamma}(i\omega)\mathbf{N}'(i\omega)[\mathbf{G}']^{-1}\int_0^\infty d\tau e^{-i\omega\tau}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iu\tau}[\mathbf{N}'(u)]^\dagger\mathbf{\Gamma}^\dagger(u)\mathbf{\Phi}_m(u)du\right\}. \tag{75}$$

From (74), it is found that

$$[\mathbf{N}'(i\omega)]^\dagger = \mathbf{K}^\dagger\mathbf{N}^\dagger(i\omega), \tag{76}$$

$$\begin{aligned}\mathbf{G}' &= [\mathbf{N}'(i\omega)]^\dagger\mathbf{\Phi}'(i\omega)\mathbf{N}'(i\omega) \\ &= \mathbf{K}^\dagger\mathbf{N}^\dagger(i\omega)\mathbf{\Phi}'(i\omega)\mathbf{N}(\omega)\mathbf{K} \\ &= \mathbf{K}^\dagger\mathbf{G}\mathbf{K},\end{aligned} \tag{77}$$

and

$$[\mathbf{G}']^{-1} = \mathbf{K}^{-1}\mathbf{G}^{-1}[\mathbf{K}^\dagger]^{-1}. \tag{78}$$

Therefore,

$$\begin{aligned}\tilde{\mathbf{H}}'(i\omega) &= \mathbf{\Gamma}(i\omega)\mathbf{N}(i\omega)\mathbf{K}\mathbf{K}^{-1}\mathbf{G}^{-1}[\mathbf{K}^\dagger]^{-1} \\ &\qquad \times \int_0^\infty d\tau e^{-i\omega\tau}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iu\tau}\mathbf{K}^\dagger\mathbf{N}^\dagger(iu)\mathbf{\Gamma}^\dagger(iu)\mathbf{\Phi}_m(iu)du\right\} \qquad (79) \\ &= \mathbf{\Gamma}(i\omega)\mathbf{N}(i\omega)\mathbf{G}^{-1}\int_0^\infty d\tau e^{-i\omega\tau}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iu\tau}\mathbf{N}^\dagger(iu)\mathbf{\Gamma}^\dagger(iu)\mathbf{\Phi}_m(iu)du\right\} \\ &= \tilde{\mathbf{H}}(i\omega).\end{aligned}$$

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SOME NEW CONCEPTS IN NONLINEAR SYSTEMS

BY

C. LAKSHMI-BAI[1]

ABSTRACT

Some new concepts characterizing the response of nonlinear systems are developed. These new concepts are denoted by the terms, the transient system equivalent, the response vector, and the space-phase components. This third concept is analyzed in comparison with the well-known technique of symmetrical components. The performance of a multiplicative feedback control system is represented by a nonlinear integro-differential equation; its solution is obtained by the principle of variation of parameters. The system response is treated as a vector and is resolved into its space-phase components. The individual effects of these components on the performance of the system are discussed. The suitability of the technique for the transient analysis of higher order nonlinear control systems is discussed.

INTRODUCTION

Of late there has been a lot of active interest displayed in the general field of nonlinear systems. Various techniques have been developed to analyze different aspects of nonlinear behavior. The phase-space technique originally developed by Cauchy (**1**)[2] and subsequently applied by Ku (**2**) to physical problems is an elegant method for the purpose of analysis. The describing function technique is a useful design tool for analyzing the system behavior under steady state. The assumptions necessary and the accuracy attainable by this method are discussed in detail by Johnson (**3**). Chen (**4**) used the principle of quasi-linearization to arrive at the transient response of the systems when the nonlinear components are of the piecewise type. Under many restrictions, a similar method has been developed by Grensted (**5**) for the transient analysis of second order nonlinear control systems.

The purpose and aim of this paper are to advance a technique which offers a new perspective and allows greater insight into the physical significance of higher order nonlinear systems. In the development of this method, based essentially on the principle of the variation of parameters, some new concepts have been recognized. These concepts, when investigated in detail, led to very interesting properties. The most important feature of the concept of space-phase components is its striking analogy to the classic technique of symmetrical components (**6**). Details of working in case of a third order nonlinear control system are illustrated, and the results are compared with those obtained by computer analysis. The concepts are extended to fourth and higher order systems.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

NOMENCLATURE

$r(t)$ = Reference input to system
$e(t)$ = instantaneous error
$C(t)$ = system response
$\psi(t)$ = instantaneous phase of oscillation
$\omega(t)$ = instantaneous frequency
$= \dfrac{d\psi}{dt}$ time rate of change of phase,
π = multiplying unit

BASIC THEORY

The solution of a nonlinear differential equation of a given order is closely related to the number and the nature of roots of the associated linear differential equation. Further, the solution of an n^{th} order differential equation contains n arbitrary parameters which can be determined by utilizing the n boundary conditions available.

Definitions of New Concepts

Let the response vector be defined as the solution of the given nonlinear differential equation. The significance of the term "response vector" is to indicate the interconnection of a physical behavior with a mathematical analysis. The term "response" itself stands for the response of the physical system, while the term "vector" stands for the corresponding mathematical analog. Thus the degrees of freedom of the physical system represented by the order of the differential equation corresponds to the number of components of the vector, or the dimensions of the space to which the vector belongs. In other words, while the dimensional space of the response vector depends on the order of the governing equation, the nature of its components varies with the type of input and the associated initial conditions.

Let the "transient system equivalent" be defined as a mathematical representation of the original nonlinear system. In such a representation there are no more cross-harmonic terms present, yet the original nonlinear effects have been adequately accounted for by their d.c. and first harmonic terms. This mathematical representation takes the form of a vector referred to an orthogonal reference frame in a space of as many dimensions as the order of the original differential equation. As will be seen later, this representation is valid in the transient as well as in the steady state.

The two concepts as defined above naturally lead to the third concept of "space-phase components." These are the projections of the response vector along the various co-ordinate axes of the orthogonal reference frame. A geometrical interpretation is possible in case of a third order system, and the concepts can be extended to the analysis of higher order systems. This to some extent explains the need for the terms "space" and "components." The term "phase" also has its physical significance, based on the choice of the reference axes, along the directions of the in-phase, the quadrature and the zero-phase components of the response vector. The in-phase is with reference to the direction, predetermined on the basis of the associated linear differential equation, while the quadrature component is at right angles to the in-phase component. The zero-phase component, on the other hand, includes all the terms which are independent of phase. Such terms may arise from the real roots of the associated linear differential equation, as well as the d-c. components due to the additional nonlinear terms. Thus, it can be easily visualized that the direction of the zero-phase component is at right angles to the plane containing the in-phase and the quadrature components. Further, the order of the zero-phase component is equal to the number of real roots of the associated linear differential equation. For example, in case of a third order system, the order of the zero-phase component can be one, or three as the case may be, while in the case of a fourth order system, it can be two or four, because complex conjugate roots have to appear in pairs.

Thus, one can correlate the degrees of freedom of the response vector with the order of the governing differential equation. Further, the nature of the space-phase components is tied up with the nature of the roots of the associated linear differential equation, modified to some extent by the nature of the non-linear terms present.

For the sake of simplicity, the following discussion is with reference to a third order system, though similar principles apply to higher order systems, with a few additional remarks, which are enumerated in later pages.

For a moment let one consider the total degrees of freedom of three vectors in a three-dimensional space. With no restrictions whatever, each of the vectors has three degrees of freedom; thus the three vectors together have nine degrees of freedom. However, let the following two restrictions be imposed. One of the three vectors is along a specified direction (determined by the phase of the assumed solution) and the other two directions are mutually orthogonal to it. Now each of such vectors has only one degree of freedom, which is influenced by the corresponding magnitude. Thus, three such vectors together have only three degrees of freedom. This statement can be easily verified with the aid of spherical coordinates. Hence, it is possible to represent the solution of a third order differential equation as a response vector in a three-dimensional space, having three mutually orthogonal components.

Analogy with Symmetrical Components

It is well known that by the method of symmetrical components, a set of unbalanced currents (or voltages) may be resolved into systems of balanced currents (or voltages), equal in number to the number of phases involved. The principle in essence is that an unbalanced multiphase electrical system can be represented by a simple mathematical analog. Among the varied advantages of this technique, those that are of interest to the present work are the stability study of power systems, determination of responses under unbalanced fault conditions, and evaluation of time constants. Table I shows the principal analogs between symmetrical components and the space-phase components, as developed in this paper. Table II shows some aspects of symmetrical

TABLE I.—*Analogs.*

	SYMMETRICAL COMPONENTS	SPACE-PHASE COMPONENTS
1	Number of phases of the electrical system	Order of the governing equation
2	Unbalance effects	Nonlinear effects
3	Mutual coupling between phases due to dis-symmetry	Presence of cross-harmonic terms
4	Positive and negative sequence components	In-phase and quadrature components
5	Zero-sequence component	Zero-phase component
6	A simple mathematical representation of an unbalanced electrical system	A simple mathematical representation of a nonlinear physical system
7	Enables stability study of power systems	Enables stability study of physical systems governed by the nonlinear differential equation
8	Valid for analysis of steady state, in its original form. Transient analysis can be effected by the modified Lyon's components	Valid for analysis of transient as well as steady state behavior
9	Various time constants of the systems can be determined	Time of rise, time of settling and other relevant time constants can be determined
10	Initial currents and voltages can be accounted for	Initial conditions can be accounted for
11	Suitable for the study of unbalanced fault conditions	Any type of input in closed form, can be studied, and instruments can be devised to control extraneous disturbances
12	Grounding phenomena associated with zero-sequence component	Zero-phase component influences only the amplitude of the response, while the in-phase and quadrature components influence both the amplitude and the frequency

TABLE II.—*Differences.*

SYMMETRICAL COMPONENTS	SPACE-PHASE COMPONENTS
1 Linear properties are assumed in the derivation of the equivalent representation	The only assumption made is that higher harmonics in the response of the nonlinear component are adequately filtered out by the linear part. This assumption is quite valid and practical with nonlinear control systems
2 Sequence currents, voltages and impedances	The only variable is the system response
3 Evaluation of sub-transient parameters	None
4 Synthesis of networks to represent faults	Synthesis of components to represent the true nonlinear effects is not possible because of the assumption made
5 Special features in the application to rotating machines	No essential difference in the application to static and dynamic nonlinear components
6 Sequence components are coplanar for any number of phases	Space-phase components are mutually orthogonal and are located in space

components for which there is no counterpart in the space-phase components.

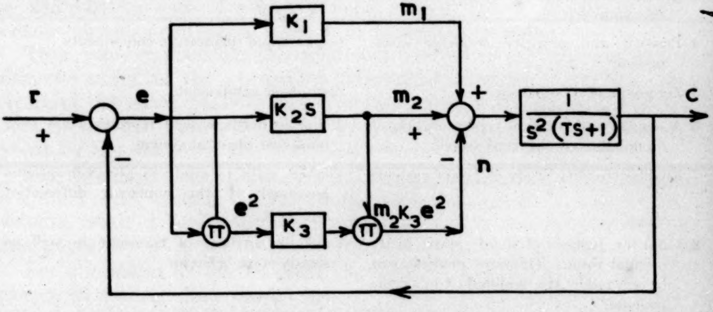


FIG. 1. Multiplicative feedback control system.

Contrast with the Phase-Space Technique

The phase-space technique portrays the relation between the system-response variable and its derivatives as a surface in a space of dimensions equal to the order of the governing equation. Though the space-phase components are also equal in number to the order of the governing differential equation, this technique represents the resolution of the response vector into mutually orthogonal components. The phase-space technique is topological in nature while the method of this paper is analytical. The phase-space technique is absolutely rigorous, while, the method of this paper is valid under one assumption (see Table II, item 1), which is practically justified with closed-loop nonlinear control systems. Thus, it can be concluded that the two methods are entirely different in their principle and application.

Illustrative Example

In Fig. 1 is shown a multiplicative feedback control system which has been studied by Ku and Chen (7). In Fig. 2 is shown the equivalent single-loop system. The governing differential equation of this system, to unit step input is given by

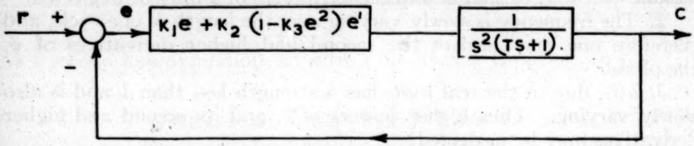


FIG. 2. Equivalent single-loop system.

$$\dddot{e} + \ddot{e} + (1 - \tfrac{1}{2}e^2)\dot{e} + \tfrac{1}{2}e = 0 \tag{1}$$

where e stands for the instantaneous error and the dots denote differentiation with respect to time. The initial conditions which the system has to satisfy are given by

$$e(0) = 0.5, \qquad \dot{e}(0) = 0, \qquad \text{and} \qquad \ddot{e}(0) = 0. \tag{2}$$

Since the principal aim of the present analysis is the application of some new concepts, only the stable case as determined by Ku and Chen is taken up for investigation. Further, transient analysis of systems is of interest only when the system is known to be stable, as determined by frequency response (8) or other preliminary design criteria. The associated linear differential equation is found to have one real root and a pair of complex conjugate roots. Therefore, one might assume, by using some engineering judgment, that the solution of Eq. 1 could be oscillatory in nature. Thus, a solution of the type shown in Eq. 3 may be assumed,

$$e(t) = a(t)\sin\psi(t) + b(t). \tag{3}$$

Equation 3 indicates that the frequency of oscillation is itself time-varying. Such, in fact, is a special feature of nonlinear systems. The actual solution in the form of Eq. 3 will be based on the principle of variation of parameters.

As a first approximation a is assumed to be a constant equal to a_0, so that the solution becomes,

$$e_1(t) = a_0 \sin \psi(t) + b(t). \tag{4}$$

Further assumptions made in the first approximation are:

1. Since $e(t)$ represents the instantaneous error, in a stable system, the amplitude of the error is decreasing with time. Hence one may assume $|a| < 1$, so that a^2 and higher powers of a may be neglected.
2. The frequency is slowly varying over the length of one cycle, and therefore one may neglect the second and higher derivatives of ψ, the phase.
3. $b(t)$, due to the real root, has a strength less than 1 and is also slowly varying. Thus higher powers of b, and its second and higher derivatives may be neglected.
4. The higher harmonics in the solution for e, or the response vector, may also be neglected, since the linear part will act as an effective low-pass filter.
5. Higher powers of $\dot{\psi}$, the frequency, may not be neglected, as it is likely to be greater than 1.

It is important to observe that the precise basis of the above assumptions is not always too clear. Their validity can only be confirmed after comparison with computer analysis. However, an experienced designer is capable of exercising this discretion, and the likely errors are expected to be within tolerable limits.

Substituting from Eq. 4 into Eq. 1, and simplifying according to the above assumptions, one gets

$$(- a_0\dot{\psi}^2 + \tfrac{1}{2}a_0 - a_0 b\dot{b}) \sin \psi + (a_0\dot{\psi} - a_0\dot{\psi}^3) \cos \psi + (\tfrac{1}{2}b + \dot{b}) = 0. \tag{5}$$

As discussed under the basic theory, coefficients of each of the three phase components in Eq. 5 can be set equal to zero, thus leading to three simultaneous equations:

$$- a_0\dot{\psi}^2 + \tfrac{1}{2}a_0 - a_0 b\dot{b} = 0 \tag{6}$$

$$- a_0\dot{\psi}^3 + a_0\dot{\psi} = 0 \tag{7}$$

$$\tfrac{1}{2}b + \dot{b} = 0. \tag{8}$$

Solution of Eqs. 7 and 8, followed by further integrations leads to

$$b(t) = b_0\epsilon^{-t/2} \tag{9}$$

$$\psi(t) = t + \text{constant}. \tag{10}$$

Because of the first approximation, Eq. 6 is of little use.

Using the initial conditions according to Eq. 2, Eq. 10 reduces to

$$\psi(t) = t + \frac{\pi}{2}. \tag{11}$$

Now, substituting in Eq. 4 from Eqs. 9 and 11, one gets

$$e_1(t) = a_0 \sin\left(t + \frac{\pi}{2}\right) + b_0\epsilon^{-t/2}. \tag{12}$$

As a second approximation, let a be a function of time $a(t)$, so that

$$e_2(t) = a(t) \sin\left(t + \frac{\pi}{2}\right) + b_0\epsilon^{-t/2}. \tag{13}$$

By physical considerations $a(t)$ may be assumed to be exponential in nature.

Thus, let

$$a(t) = a_0\epsilon^{\alpha t}. \tag{14}$$

Using Eqs. 13 and 14 along with Eqs. 1 and 2, one gets,

$$e(0) = a_0 + b_0 = 0.5 \tag{15}$$

$$\dot{e}(0) = \alpha a_0 - 0.5b_0 = 0 \tag{16}$$

$$\ddot{e}(0) = \alpha^2 a_0 - a_0 + 0.25b_0 = 0. \tag{17}$$

Solving Eqs. 15, 16 and 17 simultaneously, one gets

$$a_0 = -\tfrac{1}{6}, \quad \alpha = -2, \quad \text{and} \quad b_0 = \tfrac{2}{3}. \tag{18}$$

Now, the required solution according to Eq. 3 is given by

$$e(t) = -\tfrac{1}{6}\epsilon^{-2t} \sin\left(t + \frac{\pi}{2}\right) + \tfrac{2}{3}\epsilon^{-t/2}. \tag{19}$$

Checking for the accuracy of the assumptions made with the aid of the specified initial conditions, one finds that, $\ddot{e}(0) = -\frac{1}{3}$, instead of zero.

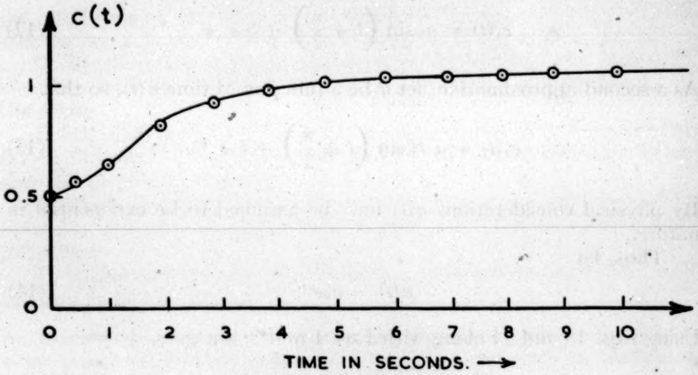
For a unit step input, Eq. 19 leads to the system response,

$$C(t) = 1 + \tfrac{1}{6}\epsilon^{-2t} \sin\left(t + \frac{\pi}{2}\right) - \tfrac{2}{3}\epsilon^{-t/2}. \tag{20}$$

As shown in Fig. 3, the evaluated analytical solution for Eq. 20, agrees very favorably with that recorded on the computer (2, p. 320, Figs. 12–24).

In the above derivation, the solution for $e(t)$ assumed on the basis of the associated linear differential equation is of the type

$$e(t) = a_0\epsilon^{\alpha t}\sin\psi(t) + b_0\epsilon^{\beta t}. \tag{21}$$



—— Response curve from analog study
⊙ Points calculated from analytical study.

FIG. 3.

Such a solution contains five parameters to be determined from the three initial conditions according to Eq. 2. The additional equations necessary are supplied by utilizing the concept of the space-phase components.

Thus the final solution is obtained by the process of successive approximations based on the principle of variation of parameters.

Comparing Eqs. 21 and 20, one finds that the zero-phase component of the response vector is given by $1 - \frac{2}{3}\epsilon^{-t/2}$, while the in-phase component is given by $\frac{1}{6}\epsilon^{-2t}\sin\left(t + \frac{\pi}{2}\right)$; the quadrature component itself is zero. It can be easily visualized that the zero-phase component is of order one and modifies only the amplitude, while the in-phase component effects both the amplitude and the frequency.

DISCUSSION

During the course of the solution, the coefficients of terms containing $\sin\psi$, $\cos\psi$ and terms independent of ψ are separately set equal to zero. From the point of view of equilibrium of forces, this stands for the fact that the components along mutually orthogonal directions of the resultant force are individually zero. This further led to the recognition of the fact that the net response vector can be resolved into space components which characterize the phase properties. It has also been observed that the response vector has already accounted for the nonlinear effects, and therefore such a resolution into space-phase components is independent of the nature of the system.

The technique developed in this paper is particularly suitable for multiplicative feedback control systems, because asymmetric and dynamic nonlinearities do not present any special difficulties. The additional advantages of the method as compared to the describing function technique are that different types of inputs in closed form can be effectively treated, along with specified initial conditions. This latter fact makes the technique particularly useful for evaluating the important transient characteristics such as the peak overshoot, time of rise, time of settling and also the possible limit-cycle behavior.

It is to be observed that with systems higher than the third order, prior engineering judgment is necessary to make an approximate guess about the several modes of oscillation and the possible number of real roots. For example, in the case of a fifth order system, there may be only one pair of complex-conjugate roots and three real roots, which means that the corresponding zero-phase component is of order three. It may also happen that the fifth order system may exhibit two modes of oscillation due to two pairs of complex-conjugate roots and the remaining one real root contributes to a zero-phase component of order one. Thus the concepts discussed with respect to third order systems can be carried over to higher order systems in analysis. However, geometric representation is no longer possible for systems of order greater than the third.

It is easily visualized from the above discussion, that the zero-phase component influences only the amplitude of the response, while the in-phase and quadrature components affect the amplitude as well as the frequency of the response. Thus it is seen that these new concepts isolate important characteristics into quantities which represent a better criterion of the controlling factors. This feature makes it a powerful tool in the synthesis of nonlinear systems.

Advantages of the New Approach

1. An approximate solution for the nonlinear differential equation can be assumed, based on the associated linear differential equation.
2. The above fact enables the use of the method of variation of parameters.
3. Successive approximations improve the degree of accuracy.

4. No special difficulties are encountered with dynamic nonlinear components.

5. Particularly suitable for multiplicative feedback control systems.

6. Mathematical representation is free from cross-harmonic terms, yet the nonlinear behavior has been adequately accounted for.

7. D-c. terms arising from nonlinear components form part of the zero-phase component; thus they can be easily taken care of.

8. Resolution into space-phase components provides the additional equations necessary for evaluating the parameters in the assumed solution.

9. Effects on the system due to the different phase components are isolated. This is a desirable feature in synthesis.

10. Suitable for transient analysis of higher order nonlinear control systems.

CONCLUSIONS

1. Three new concepts—the transient system equivalent, the response vector, and the space-phase components—are developed.

2. These new concepts are compared with the classic technique of symmetrical components.

3. An illustrative example is worked out, discussing the various aspects and advantages of this new method of solution of nonlinear systems.

4. It is believed that this technique will lead to further developments in the theory and application to nonlinear control systems, possibly even for random inputs.

Acknowledgment

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RESTRICTIONS ON THE NATURAL FREQUENCIES OF AN RC-RL NETWORK *

BY

D. A. CALAHAN [1]

ABSTRACT

The following problem is investigated: given any polynomial $N(s)$ of the form

$$N(s) = \prod_1^n (s + p_i) \prod_1^m (s + s_i)(s + \bar{s}_i)$$

what are the necessary and sufficient conditions that $N(s)$ be formed in the following manner:

$$N(s) = A_0 \Pi(s + a_i) + B_0 \Pi(s + b_i) \qquad A_0,\ B_0 > 0$$

where the $-a_i$ and $-b_i$ alternate at most in pairs along the negative real axis. Such formation is necessary for the $-p_i$ and $-s_i$ to be the natural frequencies of a network consisting of RC and RL subnetworks joined at two terminals.

It is shown that $N(s)$ may be so formed if and only if

(a) for $n = 0$, $\sum_1^m \arg s_i \leq \pi/2$

(b) for $n > 0$, $p_i > 0$ and $\sum_1^m \arg s_i < \pi/2$

Also, a simple method is presented for finding suitable a_i and b_i whenever formation is possible.

The result is expected to have application in the synthesis of both passive and active networks.

It is well known that if a network function has complex poles, then any passive realization must contain at least one inductor. DeClaris (**1**)[2] and Hakimi (**2**) have studied the natural frequencies of this "single inductor" case (Fig. 1). Horowitz (**3**) has similarly obtained restrictions for the "two-inductor" model of Fig. 2. It is now desired to obtain necessary and sufficient conditions that a given set of natural frequencies may be those of a network of the form of Fig. 3, where the RC and RL networks are of any complexity.

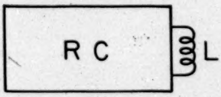

FIG. 1. Single inductor model.

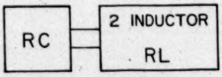

FIG. 2. Two inductor model.

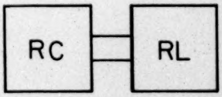

FIG. 3. RC–RL model.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

THE APPROACH TO THE PROBLEM

Let a polynomial $N(s)$ contain all the given natural frequencies. Then, for these frequencies to be those of the RC–RL model, $N(s)$ must be formed in the following manner:

$$\frac{N(s)}{D(s)} = z_{RL}(s) + z_{RC}(s) \tag{1}$$

where $z_{RL}(s)$ and $z_{RC}(s)$ are RC and RL driving point impedances,[3] respectively, and where $D(s)$ is unspecified, it being composed of the poles of $z_{RL}(s)$ and $z_{RC}(s)$. The alternation of the poles and zeros of $z_{RL}(s)$ and $z_{RC}(s)$ along the real axis imposes restrictions on the zeros of $N(s)$. The nature of these restrictions is the subject of this investigation.

To begin, Eq. 1 may be re-written

$$\frac{N(s)}{D(s)} = \frac{n_{RL}(s)}{d_{RL}(s)} + \frac{n_{RC}(s)}{d_{RC}(s)} \tag{2}$$

$$\frac{N(s)}{D(s)} = \frac{n_{RL}(s)d_{RC}(s) + n_{RC}(s)d_{RL}(s)}{d_{RL}(s)d_{RC}(s)} \tag{3}$$

$$\frac{N(s)}{D(s)} = \frac{A_{ON}A_N(s) + B_{ON}B_N(s)}{d_{RL}(s)d_{RC}(s)} \tag{4}$$

Hence,

$$N(s) = A_{ON}A_N(s) + B_{ON}B_N(s) \tag{5}$$

$$N(s) = A_{ON}\prod_1^q (s + a_i) + B_{ON}\prod_1^r (s + b_i) \tag{6}$$

$$a_i \le a_{i+1}$$
$$b_i \le b_{i+1}$$
$$a_1 \le b_1$$

A necessary condition, then, that the zeros of $N(s)$ be formed by the sum of an RC and an RL impedance is that $N(s)$ be decomposable into an $A_{ON}A_N(s)$ and a $B_{ON}B_N(s)$ whose zeros can be manipulated to form the component impedances. The following theorem will place more meaningful bounds on $A_N(s)$ and $B_N(s)$.

Theorem I: Let the number of zeros of $A_N(s)$ and $B_N(s)$ to the right of any point σ on the real axis be $Z_A(\sigma)$ and $Z_B(\sigma)$, respectively. Then

$$2 \ge Z_A(\sigma) - Z_B(\sigma) \ge 0 \qquad -\infty < \sigma < \infty \tag{7}$$

---

[3] The formation of $N(s)$ from RC and RL admittances is an identical problem.

Proof: Let the number of zeros of $n_{RL}(s)$, $n_{RC}(s)$, $d_{RL}(s)$, and $d_{RC}(s)$ to the right of any σ be $Z_{n_L}(\sigma)$, $Z_{n_c}(\sigma)$, $Z_{d_L}(\sigma)$, and $Z_{d_c}(\sigma)$, respectively. From the properties of RC and RL impedances,

$$1 \geq Z_{n_L}(\sigma) - Z_{d_L}(\sigma) \geq 0 \tag{8}$$

$$1 \geq Z_{d_c}(\sigma) - Z_{n_c}(\sigma) \geq 0. \tag{9}$$

Adding Eqs. 8 and 9, it is found

$$2 \geq Z_{n_L}(\sigma) + Z_{d_c}(\sigma) - Z_{d_L}(\sigma) - Z_{n_c}(\sigma) \geq 0. \tag{10}$$

But, from defining Eqs. 3 and 4,

$$Z_{n_L}(\sigma) + Z_{d}\ (\sigma) = Z_A(\sigma) \tag{11}$$

and

$$Z_{d_L}(\sigma) + Z_{n_c}(\sigma) = Z_B(\sigma). \tag{12}$$

From Eqs. 10, 11 and 12, then,

$$2 \geq Z_A(\sigma) - Z_B(\sigma) \geq 0. \tag{13}$$

THE OPTIMUM DECOMPOSITION

The useful necessary conditions imposed upon $A_{ON}A_N(s)$ and $B_{ON}B_N(s)$ by the properties of the RC and RL impedances are now summarized:

1. A_{ON} and B_{ON} are real and positive (14)
2. a_i and b_i are real (15)
3. $2 \geq Z_A(\sigma) - Z_B(\sigma) \geq 0$ (16)
4. $-a_i$ and $-b_i$ are non-positive. (17)

The question of whether a given $N(s)$ can be decomposed into an $A_{ON}A_N(s)$ and a $B_{ON}B_N(s)$ which fulfill Conditions 14–17 will now be considered.

The satisfaction of Condition 17 is clearly guaranteed from the non-positiveness of $-a_1$. If it were possible to find a decomposition of $N(s)$ which, of the class of all decompositions which satisfy Conditions 14–16, minimizes $-a_1$, then only the $-a_1$ of this decomposition would have to be examined to predict the possibility of RC–RL decomposition of $N(s)$. Such a decomposition will be termed the optimum decomposition and will be written $N(s) = A_1(s) + B_{01}B_1(s)$. Its form is now deduced.

Theorem II: Let $N(s)$ be a polynomial with real zeros and pairs of complex zeros, so that

$$N(s) = \prod_{1}^{n} (s + p_i) \prod_{1}^{m} (s + s_i)(s + \bar{s}_i). \tag{18}$$

Then of all the decompositions

$$N(s) = A_{ON}A_N(s) + B_{ON}B_N(s) \tag{19}$$

where $A_N(s)$ and $B_N(s)$ fulfill Conditions 14–16, the decomposition that minimizes $-a_1$ is of the form

$$A_1(s) = \prod_{1}^{n} (s + p_i) \prod_{1}^{m} (s + a_{2k-1})^2 \tag{20}$$

and

$$B_{01}B_1(s) = B_{01} \prod_{1}^{n} (s + p_i) \prod_{1}^{m-1} (s + b_{2k-1})^2. \tag{21}$$

Proof: Given any $A_{ON}A_N(s)$ and $B_{ON}B_N(s)$ which satisfy Theorem I, it will be shown that they can always be converted to $A_1(s)$ and $B_{01}B_1(s)$ while $-a_1$ is decreased or remains the same. Let

$$A_{ON}A_N(s) = A_{ON} \prod_{1}^{q} (s + a_i) \tag{22}$$

and

$$B_{ON}B_N(s) = B_{ON} \prod_{1}^{r} (s + b_i) \tag{23}$$

where

$$2 \geq q - r \geq 0 \tag{24}$$

in accordance with Theorem I. Then form

$$A_{ON+1}A_{N+1}(s) = A_{ON}A_N(s) + \delta_0 \prod_{1}^{t} (s + \delta_i) \tag{25}$$

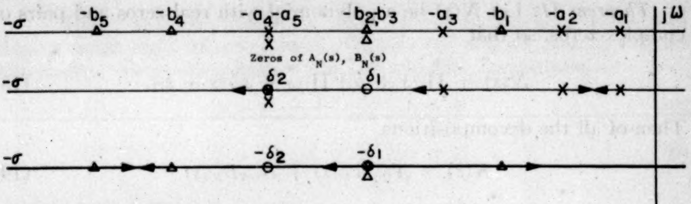
$$B_{ON+1}B_{N+1}(s) = B_{ON}B_N(s) - \delta_0 \prod_{1}^{t} (s + \delta_i) \tag{26}$$

where

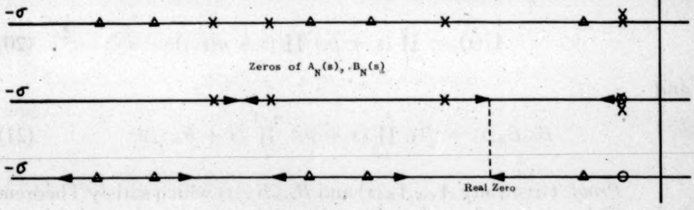
$$t \leq r \tag{27}$$

so that B_{ON+1} remains positive for some $\delta_0 > 0$, and where δ_i are located at any double zeros of $A_N(s)$ and $B_N(s)$.

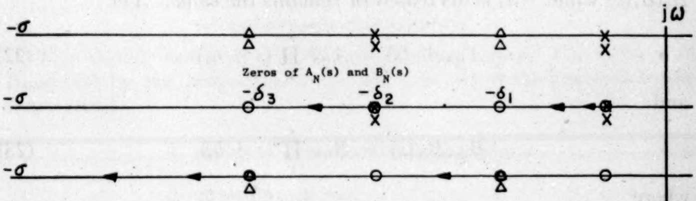
As δ_0 increases from zero, one of three things can happen which may


-σ
-b_5
-b_4
-a_4-a_5
-b_2,b_3
-a_3
-b_1
-a_2
-a_1
jω
Zeros of $A_N(s)$, $B_N(s)$
-σ
δ_2
δ_1
-σ
-δ_2
-δ_1

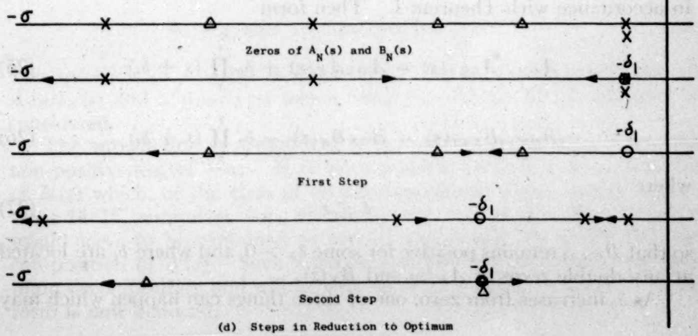
(a) Production of Double Zeros of $A_{N+1}(s)$ and $B_{N+1}(s)$


-σ
Zeros of $A_N(s)$, $B_N(s)$
-σ
-σ
Real Zero

(b) Production of Real Zero of $N(s)$


jω
-σ
Zeros of $A_N(s)$ and $B_N(s)$
-σ
-δ_3
-δ_2
-δ_1
-σ

(c) Production of Zero at Infinity


-σ
Zeros of $A_N(s)$ and $B_N(s)$
-σ
-δ_1
-σ
-δ_1
First Step
-σ
-δ_1
-σ
-δ_1
Second Step

(d) Steps in Reduction to Optimum

FIG. 4.

cause the resulting $A_{N+1}(s)$ and $B_{N+1}(s)$ to violate Theorem I. They are listed below, together with a procedure to avoid the violation.

1. For some δ_{0_1}, any zeros of $A_{N+1}(s)$ or $B_{N+1}(s)$ could become double before becoming complex (Fig. 4*a*). If this occurs, choose $\delta_0 = \delta_{0_1}$. Then by use of Eqs. 25 and 26, form a new decomposition from $A_{N+1}(s)$ and $B_{N+1}(s)$. Note that the δ_i have now changed from the original.

2. For some δ_{0_2}, real zeros of $A_{N+1}(s)$ could become equal to real zeros of $B_{N+1}(s)$ (Fig. 4*b*). If this occurs, choose $\delta_0 = \delta_{0_2}$ and repeat Eqs. 25 and 26 for

$$\frac{A_{0N+1}A_{N+1}(s)}{\prod_{1}^{n'} (s + p_i)} \quad \text{and} \quad \frac{B_{0N+1}B_{N+1}(s)}{\prod_{1}^{n'} (s + p_i)} \tag{28}$$

where $-p_i$ are the common real zeros of $A_{N+1}(s)$ and $B_{N+1}(s)$. These are, of course, real zeros of $N(s)$ also.

3. For some δ_{0_3}, if $t = r$, a zero of $B_{N+1}(s)$ could travel to $-\infty$ (Fig. 4*c*). In this case, choose $\delta_0 = \delta_{0_3}$ and repeat Eqs. 25 and 26 for the resultant $A_{N+1}(s)$ and $B_{N+1}(s)$.

In any of steps 1, 2, or 3 it is clear that no zeros of $A_{N+1}(s)$ can occur to the right of $-a_1$ of $A_N(s)$ since this region is void of any zeros of $\delta(s)$. Therefore, $-a_1$ must be decreased or, at worst, remain the same. The latter will happen only when a double zero of $A_N(s)$ occurs at $-a_1$. The next step must then decrease $-a_1$, as Fig. 4*d* demonstrates.

For any decomposition to minimize $-a_1$, it must be immune to improvement by all of the preceding steps. But the only assumption made in forming new decompositions is $t \leq r$, which therefore must not be satisfied by the optimum decomposition.

Now $t > r$ requires that there be more pairs of double zeros of $A_N(s)$ and $B_N(s)$ than there are zeros of $B_N(s)$. This does not include the real zeros of $N(s)$ that have been omitted from the process by Step 2. Let

$$q = 2q_1 + q_2 \tag{29}$$

$$r = 2r_1 + r_2 \tag{30}$$

where q_1 and r_1 are the number of pairs of double zeros of $A_N(s)$ and $B_N(s)$ and where q_2 and r_2 are the number of their simple zeros, respectively. Then $t > r$ demands

$$q_1 + r_1 > 2r_1 + r_2 \tag{31}$$

or

$$q_1 - r_1 > r_2. \tag{32}$$

Also, from Eq. 24

$$2 \geq 2q_1 + q_2 - 2r_1 - r_2 \tag{33}$$

$$\geq 2(q_1 - r_1) + q_2 - r_2. \tag{34}$$

Combining Eqs. 32 and 34, it follows that

$$r_2 + q_2 < 2. \tag{35}$$

From Eq. 35 several possibilities are suggested. If $r_2 = 1$ and $q_2 = 0$, then Eqs. 32 and 33 become

$$1 < q_1 - r_1 \leq \tfrac{3}{2}. \tag{36}$$

If $r_2 = 0$ and $q_2 = 1$, then

$$0 < q_1 - r_1 \leq \tfrac{1}{2}. \tag{37}$$

Neither Eq. 36 nor 37 can be satisfied by integral values of q_1 and r_1, so that the assumed values of r_2 and q_2 are impossible. On the other hand, if $r_2 = 0$ and $q_2 = 0$, then $0 < q_1 - r_1 \leq 1$, which can be satisfied if $q_1 = r_1 + 1$. Therefore, the final decomposition must have all double zeros ($q_2 = 0$, $r_2 = 0$) with $q = r + 2$. But if $A_N(s)$ and $B_N(s)$ have only double zeros, their sum can realize only complex zeros. All the real zeros of $N(s)$, then, must have been deleted by Step 2 in the procedure. Therefore, the optimum decomposition has the form

$$N(s) = \prod_1^n (s + p_i)\left[\prod_1^m (s + a_{2k-1})^2 + B_{01} \prod_1^{m-1} (s + b_{2k-1})^2\right] \tag{38}$$

where the a_{2k-1} and b_{2k-1} alternate since the optimum must satisfy Theorem I. Equation 38 is of the same form as Eqs. 20 and 21 so the theorem is proved.

THE EXISTENCE OF THE OPTIMUM

The existence of the optimum decomposition will now be established by showing that it may be simply formed from $N(s)$. Since the real zeros of $N(s)$ are separate from the remainder of the decomposition (Eq. 38), the only question is whether the complex zeros may be decomposed as shown.

Theorem III: Any polynomial of the form

$$N(s) = \prod_1^m (s + s_i)(s + \bar{s}_i) \tag{39}$$

can be decomposed into the form

$$N(s) = A_1(s) + B_{01}B_1(s) \tag{40}$$

where

$$A_1(s) = \prod_1^m (s + a_{2k-1})^2 \tag{41}$$

and

$$B_{01}B_1(s) = B_{01} \prod_1^{m-1} (s + b_{2k-1})^2 \tag{42}$$

where

1. B_{01} is positive and real, (43)
2. a_{2k-1} and b_{2k-1} are real, (44)
3. $a_1 < b_1 < a_3 < b_3 < \cdots < a_{2m-1}$. (45)

Proof: Let

$$A_1(s) = [R_m(s)]^2 \tag{46}$$

and

$$B_{01}B_1(s) = [Q_m(s)]^2 \tag{47}$$

so that

$$R_m(s) = \prod_1^m (s + a_{2k-1}) \tag{48}$$

and

$$Q_m(s) = \sqrt{B_{01}} \prod_1^{m-1} (s + b_{2k-1}). \tag{49}$$

Then

$$N(s) = [R_m(s)]^2 + [Q_m(s)]^2 \tag{50}$$

$$= [R_m(s) + jQ_m(s)][R_m(s) - jQ_m(s)]. \tag{51}$$

Now consider $N(s)$. Its factors are of the form

$$N(s) = [\prod_1^m (s + c_i + jd_i)][\prod_1^m (s + c_i - jd_i)] \tag{52}$$

where c_i are real and $d_i > 0$. Comparing Eqs. 51 and 52, the following identifications can be made

$$R_m(s) + jQ_m(s) = \prod_1^m (s + c_i + jd_i) \tag{53}$$

$$R_m(s) - jQ_m(s) = \prod_1^m (s + c_i - jd_i). \tag{54}$$

It must now be shown that the coefficients and zeros of $R_m(s)$ and $Q_m(s)$ satisfy Conditions 43–45.

First consider the case $m = 2$.

$$R_2(s) + jQ_2(s) = (s + c_1 + jd_1)(s + c_2 + jd_2) \tag{55}$$

$$\begin{aligned} &= (s + c_1)(s + c_2) - d_1d_2 \\ &\qquad + j[d_1(s + c_2) + d_2(s + c_1)]. \end{aligned} \tag{56}$$

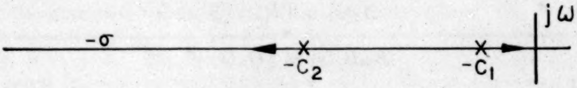
Hence,

$$R_2(s) = (s + c_1)(s + c_2) - d_1d_2 \tag{57}$$

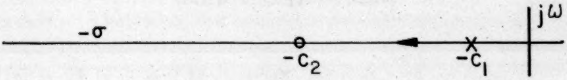
and

$$Q_2(s) = d_1(s + c_2) + d_2(s + c_1). \tag{58}$$

The root locus of $R_2(s)$ and $Q_2(s)$ as d_1 and d_2 are varied is shown in Fig. 5.



Root Locus of $R_2(s)$



Root Locus of $Q_2(s)$

FIG. 5.

The coefficients of the highest power of s in $R_2(s)$ and $Q_2(s)$ are 1 and $d_1 + d_2$, respectively, both of which are positive and real. Moreover, the zeros of $R_2(s)$ and $Q_2(s)$ are shown by Fig. 5 to be real and to alternate along the real axis. Therefore the theorem is proved for $m = 2$.

For $m > 2$, suppose, in the process of forming $R_m(s)$ and $Q_m(s)$ by multiplication of terms like $s + c_i + jd_i$, the r^{th} multiplication is being performed. Then

$$R_r(s) + jQ_r(s) = (s + c_r + jd_i)\left[\prod_1^{r-1} (s + \alpha_i) + jK_{r-1} \prod_1^{r-2} (s + \beta_i)\right] \tag{59}$$

$$R_r(s) + jQ_r(s) = \left[(s + c_r)\prod_1^{r-1}(s + \alpha_i) - d_rK_{r-1}\prod_1^{r-2}(s + \beta_i)\right]$$

$$+ j\left[d_r\prod_1^{r-1}(s + \alpha_i) + (s + c_r)K_{r-1}\prod_1^{r-2}(s + \beta_i)\right] \quad (60)$$

$$= \prod_1^{r}(s + \alpha_i') + jK_r\prod_1^{r-1}(s + \beta_i'). \quad (61)$$

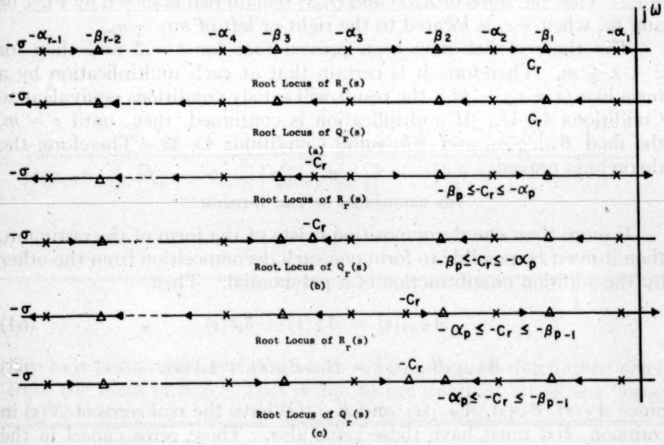

jω
-σ
$-\alpha_{r-1}$
$-\beta_{r-2}$
$-\alpha_4$
$-\beta_3$
$-\alpha_3$
$-\beta_2$
$-\alpha_2$
$-\beta_1$
$-\alpha_1$
C_r
Root Locus of $R_r(s)$
C_r
Root Locus of $Q_r(s)$
(a)
$-C_r$
$-\beta_p \leq -C_r \leq -\alpha_p$
Root Locus of $R_r(s)$
$-C_r$
$-\beta_p \leq -C_r \leq -\alpha_p$
Root Locus of $Q_r(s)$
(b)
$-C_r$
$-\alpha_p \leq -C_r \leq -\beta_{p-1}$
Root Locus of $R_r(s)$
$-C_r$
$-\alpha_p \leq -C_r \leq -\beta_{p-1}$
Root Locus of $Q_r(s)$
(c)

FIG. 6.

It will now be shown that, if K_{r-1}, α_i, and β_i satisfy conditions equivalent to Conditions 43–45 then K_r, α_i', and β_i' do also.

Consider the root locus equations of $R_r(s)$ and $Q_r(s)$:

$$\frac{R_r(s)}{(s + c_r)\prod_1^{r-1}(s + \alpha_i)} = 1 - \frac{d_rK_{r-1}\prod_1^{r-2}(s + \beta_i)}{(s + c_r)\prod_1^{r-1}(s + \alpha_i)} \quad (62)$$

$$\frac{Q_r(s)}{d_r\prod_1^{r-1}(s + \alpha_i)} = 1 + \frac{K_{r-1}(s + c_r)\prod_1^{r-2}(s + \beta_i)}{d_r\prod_1^{r-1}(s + \alpha_i)}. \quad (63)$$

Now $d_r > 0$ and $K_{r-1} > 0$ by assumption that the latter satisfied Condition 43. Since the alternation of α_i and β_i along the real axis is also assumed, the form of the root loci of $R_r(s)$ and $Q_r(s)$ can be shown (Fig. 6). In Fig. 6*a*, the zero at $-c_r$, which may lie anywhere on the real axis, is assumed at infinity. Clearly the zeros of $R_r(s)$ and $Q_r(s)$ alternate in this case. If now the zero at $-c_r$ is brought from infinity along the negative real axis, it will not affect the permitted regions of "closed loop" zeros to the right of it. To the left of it, $-c_r$ will reverse the permitted locations of the zeros of $R_r(s)$ and $Q_r(s)$. Therefore, alternation of the zeros of $R_r(s)$ and $Q_r(s)$ is preserved in the presence of $-c_r$. That the zeros of $R_r(s)$ and $Q_r(s)$ remain real is shown by Figs. 6*b* and 6*c*, when $-c_r$ is located to the right or left of any $-\alpha_p$.

The theorem has now been proved first for $r = 2$ and then for $2 < r \leq m$. Therefore, it is certain that at each multiplication by a term like $(s + c_i + jd_i)$, the result will satisfy conditions equivalent to Conditions 43–45. If multiplication is continued, then, until $r = m$, the final B_{01}, $-a_i$, and $-b_i$ fulfill Conditions 43–45. Therefore, the theorem is proved.

THE UNIQUENESS OF THE OPTIMUM

If more than one decomposition exists of the form of the optimum, then it must be possible to form one such decomposition from the other by the addition or subtraction of a polynomial. Then

$$A_{N+1}(s) = A_N(s) \pm \delta_0\delta(s) \tag{64}$$

$$B_{ON+1}B_{N+1}(s) = B_{ON}B_N(s) \mp \delta_0\delta(s). \tag{65}$$

Since $A_N(s)$, $B_N(s)$, $A_{N+1}(s)$, and $B_{N+1}(s)$ have the real zeros of $N(s)$ in common, $\delta(s)$ must have these zeros also. These zeros cancel in the above equations and may be ignored from further consideration. The equations may then be written

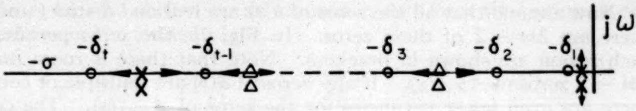
$$\prod_1^m (s + a'_{2k-1})^2 = \prod_1^m (s + a_{2k-1})^2 + \delta_0\delta(s) \tag{66}$$

$$B_{ON+1} \prod_1^{m-1} (s + b'_{2k-1})^2 = B_{ON} \prod_1^{m-1} (s + b_{2k-1})^2 - \delta_0\delta(s). \tag{67}$$

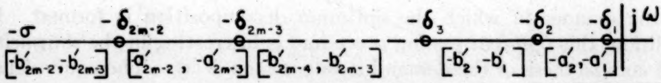
The sign preceding δ_0 may be fixed since the decomposition identified with the left hand side of these equations is arbitrary and may be identified as the one for which the above sign choice is correct. If

$$\delta(s) = \prod_1^t (s + \delta_i) \tag{68}$$

then $t \leq 2m - 2$ or else B_{ON+1} will be negative. Now if the root locus of Eq. 67 is examined (Fig. 7*a*) as δ_0 varies, the regions along the nega-



(a) Root Locus of $A_{N+1}(s)$ and $B_{N+1}(s)$



(b) Regions Permitted for Zeros

FIG. 7.

tive axis permitted for the zeros at $-b'_{2k-1}$ may be determined solely from the zeros of $\delta(s)$. This is due to the duplicity of all the "open-loop" zeros at $-b_{2k-1}$. For if the root locus condition

$$\arg \frac{\delta(s)}{\prod_{1}^{m-1} (s + b_{2k-1})^2} = n\pi \qquad n = 0, \pm 2, \pm 4, \cdots \tag{69}$$

is satisfied to the immediate right of $-b_{2k-1}$ then it will be satisfied to the immediate left of $-b_{2k-1}$. Likewise, if Eq. 69 is not satisfied to the immediate right of $-b_{2k-1}$, then neither will it be satisfied to the left. Similarly, the regions permitted for $-a'_{2k-1}$ are determined from the satisfaction of the equation

$$\arg \frac{\delta(s)}{\prod_{1}^{m} (s + a_{2k-1})^2} = n\pi \qquad n = \pm 1, \pm 3, \pm 5, \cdots. \tag{70}$$

Again, the satisfaction of this equation on the negative real axis depends solely on the zeros of $\delta(s)$. However, since the values of n that may satisfy Eqs. 69 and 70 are different, the regions permitted for the zeros at $-a'_{2k-1}$ and $-b'_{2k-1}$ do not overlap.

Now assume that all the zeros of $\delta(s)$ are real and distinct and that there are $2m - 2$ of these zeros. In Fig. 7*b*, the zeros permitted in each region are shown in brackets. Note that there is room for only $2m - 2$ zeros of $A_{N+1}(s)$. If any zeros of $\delta(s)$ are multiple or complex, there are even fewer vacancies for the zeros of $A_{N+1}(s)$. The conclusion is that no $\delta(s)$ exists which permits formation of an $A_{N+1}(s)$ of the form shown. The uniqueness of the optimum decomposition follows.

THE ANGLE CRITERION

To test whether a polynomial $N(s)$ could be formed as in Eq. 2, $A_1(s)$ and $B_{01}B_1(s)$ may be formed and $-a_1$ examined. The non-positiveness of $-a_1$ is then a necessary condition that $N(s)$ be so formed.

Even so, there is an easier method of testing $N(s)$ which is related to the manner in which the optimum decomposition is formed. In addition, this different testing procedure is interesting in the simplicity and compactness of the demand it imposes on $N(s)$. The equivalence of the two testing methods is shown by the following theorem.

Theorem IV: For any $N(s)$ of the form of Eq. 39, a necessary and sufficient condition that $-a_1$ be non-positive is[4]

$$\sum_1^m \arg s_i \leq \frac{\pi}{2}. \tag{71}$$

Proof: For any $s_i = c_i + jd_i$, define

$$\omega_i(\sigma) = \sqrt{d_i^2 + (c_i + \sigma)^2} \tag{72}$$

$$\theta_i(\sigma) = \tan^{-1} \frac{d_i}{c_i + \sigma} \qquad 0 \leq \theta_i(\sigma) \leq \pi. \tag{73}$$

The graphical interpretation of the above quantities is shown in Fig. 8*a*. Now define

$$y(\sigma) = \sum_1^m \theta_i(\sigma) \tag{74}$$

so that it must now be shown that

$$\sum_1^m \arg s_i = \sum_1^m \theta_i(0) = y(0) \leq \frac{\pi}{2} \Longleftrightarrow -a_1 \leq 0. \tag{75}$$

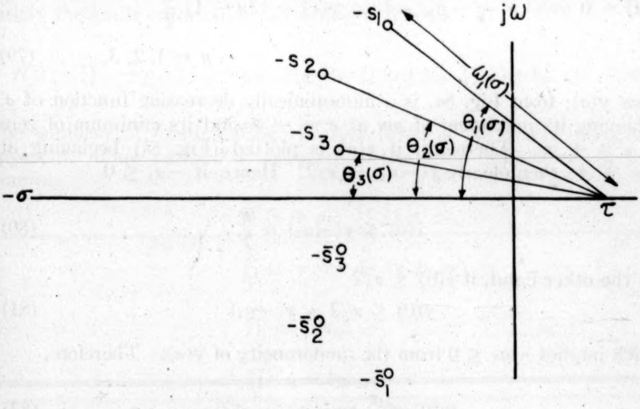
---

[4] This condition is identical with the results of Horowitz (3) for a fourth order polynomial.

Let the s-plane be shifted by an amount σ, and form the optimum decomposition in the shifted plane.

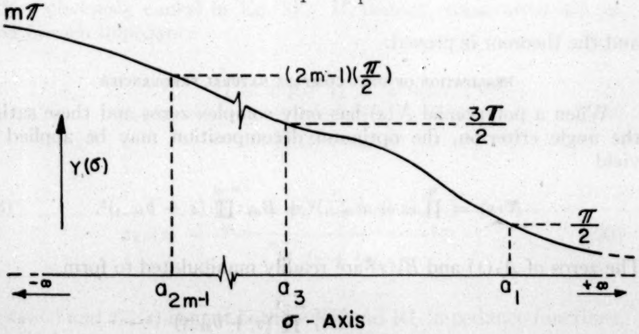
$$R_m(s+\sigma) + jQ_m(s+\sigma) = \prod_1^m (s + c_i + \sigma + jd_i) \tag{76}$$

$$= \prod_1^m [s + \omega_i(\sigma) \quad \underline{/\theta_i(\sigma)}]. \tag{77}$$



$-s_1$
$-s_2$
$-s_3$
$j\omega$
$\omega_1(\sigma)$
$\theta_1(\sigma)$
$\theta_2(\sigma)$
$\theta_3(\sigma)$
$-\sigma$
τ
$-\bar{s}_3$
$-\bar{s}_2$
$-\bar{s}_1$

(a) Interpretation of $\omega_i(\sigma)$, $\theta_i(\sigma)$



(b) Sketch of $y(\sigma)$

FIG. 8.

The constant term of $R_m(s+\sigma)$ is

$$r(\sigma) = \prod_1^m (a_{2k-1}+\sigma) = \prod_1^m \omega_i(\sigma)\cos\sum_1^m \theta_i(\sigma)$$

$$= \prod_1^m \omega_i(\sigma)\cos y(\sigma). \tag{78}$$

Therefore

$$r(\sigma) = 0 \Longleftrightarrow \sigma = -a_{2k-1} \Longleftrightarrow y(\sigma) = (2n-1)\frac{\pi}{2}$$

$$n = 1, 2, 3, \cdots. \tag{79}$$

Now $y(\sigma)$, from Fig. 8*a*, is a monotonically decreasing function of σ, achieving its maximum of $m\pi$ at $\sigma = -\infty$ and its minimum of zero at $\sigma = +\infty$. Therefore, if $y(\sigma)$ is plotted (Fig. 8*b*) beginning at $\sigma = +\infty$, then clearly $y(-a_1) = \pi/2$. Hence, if $-a_1 \leq 0$

$$y(0) \leq y(-a_1) = \frac{\pi}{2}. \tag{80}$$

On the other hand, if $y(0) \leq \pi/2$

$$y(0) \leq \pi/2 = y(-a_1) \tag{81}$$

which implies $-a_1 \leq 0$ from the monotoneity of $y(\sigma)$. Therefore,

$$y(0) \leq \frac{\pi}{2} \Longleftrightarrow -a_1 \leq 0 \tag{82}$$

and the theorem is proved.

REALIZATION OF ONLY COMPLEX NATURAL FREQUENCIES

When a polynomial $N(s)$ has only complex zeros and these satisfy the angle criterion, the optimum decomposition may be applied to yield

$$N(s) = \prod_1^m (s+a_{2k-1})^2 + B_{01}\prod_1^{m-1}(s+b_{2k-1})^2. \tag{83}$$

The zeros of $A_1(s)$ and $B_1(s)$ are readily manipulated to form

$$z_{RC}(s) = \frac{\sqrt{B_{01}}\prod_1^{m-1}(s+b_{2k-1})}{\prod_1^m (s+a_{2k-1})} \tag{84}$$

and

$$z_{RL}(s) = \frac{\prod_{1}^{m} (s + a_{2k-1})}{\sqrt{B_{01}} \prod_{1}^{m-1} (s + b_{2k-1})} \tag{85}$$

which are RC and RL driving point impedance functions.

REALIZATION OF REAL AND COMPLEX NATURAL FREQUENCIES

The optimum decompositon of an $N(s)$, the complex zeros of which satisfy the angle criterion, is, according to Theorem II,

$$N(s) = \prod_{1}^{n} (s+p_i) \prod_{1}^{m} (s+a_{2k-1})^2 + B_{01} \prod_{1}^{n} (s+p_i) \prod_{1}^{m-1} (s+b_{2k-1})^2. \tag{86}$$

The problem of manipulating the real zeros of $N(s)$ now becomes apparent. They cannot be assigned to the same impedance, as for example

$$z_{RC}(s) = \frac{\prod_{1}^{n} (s + p_i) \prod_{1}^{m-1} (s + b_{2k-1})}{\prod_{1}^{n} (s + p_i) \prod_{1}^{m} (s + a_{2k-1})} \tag{87}$$

$$z_{RL}(s) = \frac{\prod_{1}^{m} (s + a_{2k-1})}{\prod_{1}^{m-1} (s + b_{2k-1})} \tag{88}$$

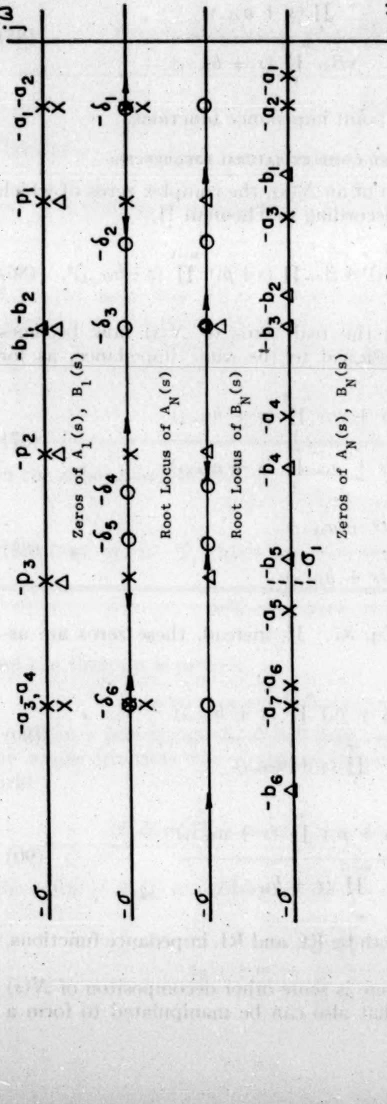
for they obviously cancel in Eq. 87. If, instead, these zeros are assigned to each impedance

$$z_{RC}(s) = \frac{\prod_{1}^{n} (s + p_i) \prod_{1}^{m-1} (s + b_{2k-1})}{\prod_{1}^{m} (s + a_{2k-1})} \tag{89}$$

$$z_{RL}(s) = \frac{\prod_{1}^{n} (s + p_i) \prod_{1}^{m} (s + a_{2k-1})}{\prod_{1}^{m-1} (s + b_{2k-1})} \tag{90}$$

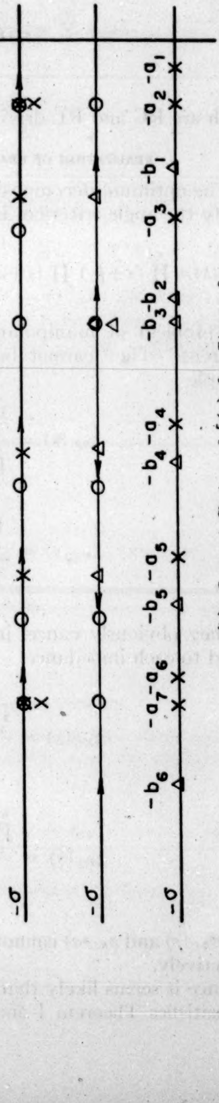
then $z_{RC}(s)$ and $z_{RL}(s)$ cannot both be RC and RL impedance functions, respectively.

Since it seems likely that there is some other decompositon of $N(s)$ that satisfies Theorem I and that also can be manipulated to form a



jω
-σ
$-a_3, -a_4$
$-p_3$
$-p_2$
$-b_1, -b_2$
$-p_1$
$-a_1 -a_2$
Zeros of $A_1(s)$, $B_1(s)$
$-\delta_6$
$-\delta_5$
$-\delta_4$
$-\delta_3$
$-\delta_2$
$-\delta_1$
Root Locus of $A_N(s)$
Root Locus of $B_N(s)$
$-b_6$
$-a_7 -a_6$
$-a_5$
$-b_5$
$-\sigma_1$
$-b_4$
$-a_4$
$-b_3 -b_2$
$-a_3 -b_1$
$-a_2 -a_1$
Zeros of $A_N(s)$, $B_N(s)$

(a) Violation of Theorem I by Incorrect $\delta(s)$



jω
-σ
$-b_6$
$-a_7 -a_6$
$-b_5 -a_5$
$-b_4 -a_4$
$-b_3 -b_2$
$-a_3 -b_1$
$-a_2 -a_1$

(b) Correct Choice of $\delta(s)$ for Figure 9a

FIG. 9*a–b*.

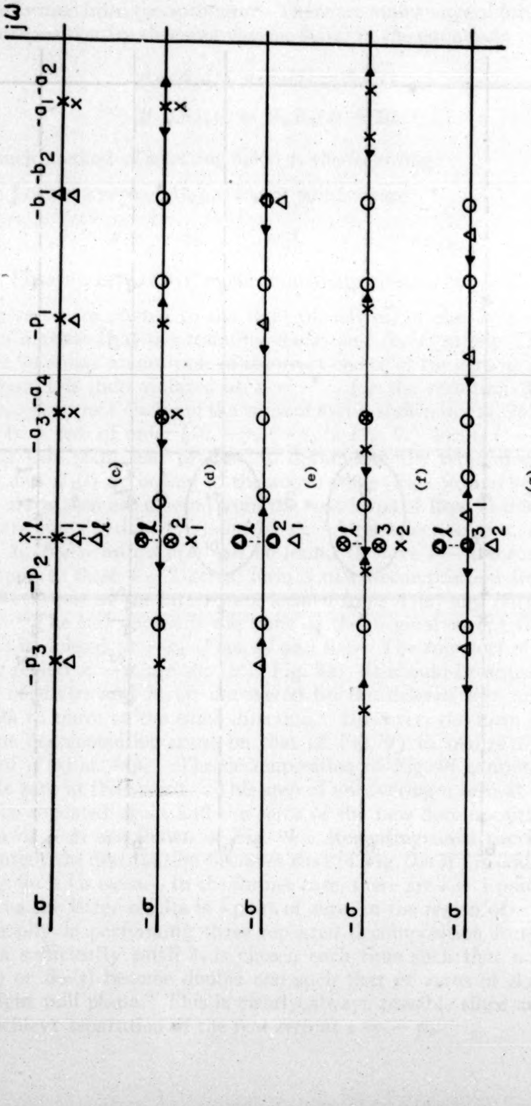

jω
-σ
-p3
-p2
-a3,-a4
-p1
-b1,-b2
-a1,-a2
l
1
2
3
(c)
(d)
(e)
(f)
(g)

FIG. 9c–h. Separation of multiple zeros.

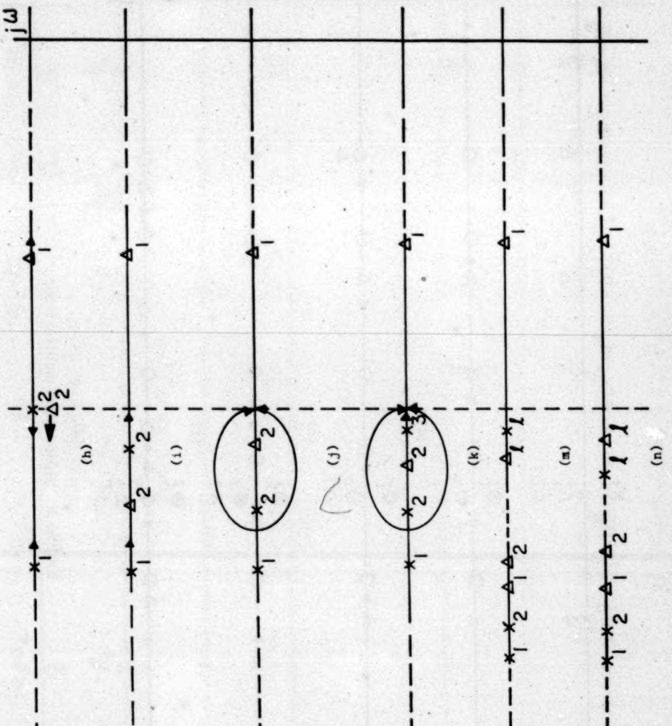

jω
-σ
1
2
3
(h)
(i)
(j)
(k)
(m)
(n)

FIG. 9h–n. Separation of multiple zeros.

$z_{RC}(s)$ and a $z_{RL}(s)$, the question arises as to how this decomposition can be formed from the optimum. There are many ways of forming such a decomposition by choosing various $\delta_0\delta(s)$ in the equations

$$A_{ON}A_N(s) = A_1(s) - \delta_0\delta(s) \tag{91}$$

$$B_{ON}B_N(s) = B_{01}B_1(s) + \delta_0\delta(s). \tag{92}$$

One such method of selecting $\delta_0\delta(s)$ is the following:

1. Locate a zero of $\delta(s)$ at every point where

$$s = -a_{2k-1} \quad \text{and} \quad s = -b_{2k-1}.$$

2. Locate a zero of $\delta(s)$ in the immediate vicinity of every $s = -p_i$.

These zeros are chosen to the right of, left of, or else at $s = -p_i$ in such a manner that the resulting $A_N(s)$ and $B_N(s)$ satisfy Theorem I. Figure 9*a* shows an example of incorrect choice of the zero at $s = -\delta_5$. Theorem I is then violated at $s = -\sigma_1$ for the resulting $A_N(s)$ and $B_N(s)$. A correct choice of the zeros of $\delta(s)$ is shown in Fig. 9*b*. Should there be a zero of order l at $-p_i$ ($-p_2$ in Fig. 9*c*), locate $l - 1$ zeros of $\delta(s)$ at this point and proceed to decompose the remaining zeros of $A_1(s)$ and $B_1(s)$ according to the above steps (Figs. 9*d* and 9*e*). These $l - 1$ zeros then are deleted from the root locus of Eqs. 91 and 92 since they are contained in $A_1(s)$ and $B_1(s)$. After selection of a sufficiently small δ_0, $B_N(s)$ and $A_N(s)$ will be found to have $l - 1$ zeros at $-p_2$. To separate these $l - 1$ zeros, form a new decomposition from $A_N(s)$ and $B_N(s)$ just as the latter were formed from $A_1(s)$ and $B_1(s)$, respectively. The $\delta(s)$ chosen is the same as the original except that a zero of $\delta(s)$ is deleted at $-p_2$ (Figs. 9*f* and 9*g*). The root loci of this step in the region of $-p_2$ are shown in Fig. 9*h*. It should be noted that the zeros of $A_N(s)$ and $B_N(s)$ uncovered by the deleted zero are seen in Fig. 9*h* to move in the same direction. However, the form of the resultant decomposition must be that of Fig. 9*j* to realize the double zero of $N(s)$ at $-p_2$. The decomposition of Fig. 9*i* cannot realize a double zero at that point. This step of uncovering a zero at $s = -p_2$ may be repeated again and the zeros of the new decomposition in the region of $-p_2$ are shown in Fig. 9*k*. Repeating until the l^{th} zero is separated, the distribution becomes that of Fig. 9*m* if l is odd and that of Fig. 9*n* if l is even. In the former case, there are $l - 1$ pairs of zeros whereas the latter results in l pairs of zeros in the region of $-p_2$. The philosophy in performing these repeated decomposition formations is that a sufficiently small δ_0 is chosen each time such that no zeros of $A_N(s)$ or $B_N(s)$ become double and such that no zeros of $A_N(s)$ enter the right half plane. This is clearly always possible since any $\delta_0 > 0$ will achieve separation of the real zero at $s = -p_2$.

The resultant $A_N(s)$ must always, regardless of $\delta(s)$, have a zero more positive than the most positive zero of $A_1(s)$. Theorem II has shown that $A_N(s)$ can always be reduced to $A_1(s)$ while decreasing the most positive zero of $A_N(s)$. Therefore, if $A_1(s)$ has a zero at $s = 0$, $A_N(s)$ must contain a zero in the right half plane. It is necessary, then, that $\sum_1^m \arg s_i < \pi/2$ and also that all the real zeros of $N(s)$ be negative in order to achieve an impedance decomposition. These conditions are sufficient as well as necessary since the optimum decomposition of any $N(s)$ satisfying them can be converted to an acceptable decomposition by choice of an arbitrarily small δ_0 in Eqs. 91 and 92.

An Example

The polynomial

$$N(s) = (s^2 + 2s + 2)(s^2 + \sqrt{3}s + 1) \tag{93}$$

is to be formed as in Eq. 1.

Separating the upper half plane zeros, one identifies

$$R(s) + jQ(s) = (s + 1 + j1)(s + .867 + j.5) \tag{94}$$

$$= (s + s_1)(s + s_2). \tag{95}$$

Here, $\arg s_1 + \arg s_2 = 5\pi/12$ so that RC–RL decomposition is possible. Further,

$$R(s) + jQ(s) = (s + .226)(s + 1.64) + j1.5(s + .91) \tag{96}$$

so that

$$A_1(s) + B_{01}B_1(s) = (s + .226)^2(s + 1.64)^2 + 2.25(s + .91)^2. \tag{97}$$

The desired formation of $N(s)$ is then

$$\frac{N(s)}{D(s)} = \frac{(s + .226)(s + 1.64)}{1.5(s + .91)} + \frac{1.5(s + .91)}{(s + .226)(s + 1.64)}. \tag{98}$$

CONCLUSIONS

In summary, the necessary and sufficient conditions that a given polynomial $N(s)$ be formed from the sum of RC and RL driving point impedances are stated:

1. If $N(s) = \prod_1^m (s + s_i)(s + \bar{s}_i)$, then $\prod_1^m \arg s_i \leq \frac{\pi}{2}$ (99)

2. If $N(s) = \prod_1^n (s + p_i) \prod_1^m (s + s_i)(s + \bar{s}_i)$, then:

$$a.\ \prod_{1}^{m} \arg s_i < \frac{\pi}{2} \tag{100}$$

$$b.\ p_i > 0. \tag{101}$$

The sufficiency of the above conditions may be expected to have application in both passive and active network synthesis, where the partitioning of network functions frequently leads to simplified models.

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SPACE, TIME, AND ETERNITY

BY

GUSTAF STROMBERG[1]

According to the theory of relativity, measurements of space and time intervals (determined with yard sticks and clocks) are dependent on one another in such a way that, mathematically, they can be represented as being "projections" of a four-dimensional world-frame called *space-time*. This term was introduced in 1908 by the famous German physicist H. Minkowski, who pictured space-time as a "ghostly" reference frame in an external world in which all physical phenomena were displayed. According to Kant, space and time are *a priori* forms of human perceptions and are apparently the product of the sensory nerve system in the human body. Of the greatest importance for our knowledge of the world around us are our sense of vision and our sense of the flow and duration of time which have enabled us to form ideas about the existence of external objects and of their motions. The science of physics has been developed for the purpose of co-ordinating our space and time measurements so as to furnish a logically consistent set of rules, generally called "laws of nature," governing these measurements. Minkowski's four-dimensional space-time was used with great success by Einstein in his general theory of relativity, to formulate laws of motion of greater generality and exactness than those formulated by Newton.

But, there are many phenomena in our experience which cannot be expressed in terms of measurements of space and time intervals. Because they are not governed by the laws of physics, such phenomena may be called *nonphysical*. A simple example is the moving pictures we see in a dream or an hallucination. Because of their private and personal nature they are classified as *subjective*, and are therefore regarded as having no objective reality. Such pictures, however, are built of the same kind of "mental stuff" as are the pictures we see with our eyes when we are fully awake and not under the influence of hypnotic suggestion or drugs.

In spite of the great advance in our knowledge of the physical world of space and time in recent years, little progress has been made in the field of research which deals with the relationship between the physical and the nonphysical worlds, sometimes referred to as the relationship between mind and matter, or, in popular language, between our body and our "soul."

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The world of matter and the world of mind appear at first sight so different from one another that it seems impossible to find any connecting links between them. In attempting to explain their relationship, some philosophers have tried to describe consciousness as an "epiphenomenon," that is, as a *by-product* associated with the stimulation of sensitive nerve cells in the cortex of the brain. Some philosophers claim that what we call "the physical world" is a play of mental "shadows," seen directly or pictured in our imagination, which we must be careful not to identify with ultimate reality, which itself may well be completely beyond human comprehension. Some scientists claim that the *rules* governing the mental shadow-play represent a more fundamental level of truth than do the individual sense data.

To represent the relationship between quantities of different nature, it is often convenient to plot them as co-ordinates in a reference frame with axes perpendicular to one another. For instance, the path of a moving object in the space-time world can be pictured as a continuous line in a four-dimensional reference frame. The point where two such "world lines" intersect represents an observable "event" in the physical world of space and time. A *mental* event, on the other hand, takes place at a certain time, but not at any particular location. Therefore, to include mental events in our world picture, we must add to the four dimensions of space-time a new "dimension" or "axis" perpendicular to the three space axes and to the time axis. (Mathematically, there is no difficulty in having as many independent dimensions in our world picture as we think are needed.)

The idea of a *five-dimensional universe* has been found very useful in theoretical physics. The German physicist A. Kaluza (**1**)[2] showed in 1921 that a unified field theory could be developed if a fifth dimension were added to the four-dimensional world of Minkowski and Einstein, and similar conclusions have been reached by other theoretical physicists. Of special interest is the fact that Prince Louis de Broglie, the prominent French physicist who in 1924 introduced the idea that all material particles must be associated with some kind of "waves," called attention to the fact that the hypothesis of a five-dimensional world-frame was a logical generalization of the relativity theory, and that it involved the assumption that the human sense organs are incapable of directly perceiving any effect due to the existence of the fifth dimension, and that what we actually perceive must be the "projections" onto the space-time world of events occurring in an extra-physical world, that is, a world beyond physical space and physical time (**2**).

An important step in the change-over from a four-dimensional to a five-dimensional universe was taken by the prominent British scientist and philosopher, J. G. Bennett. In collaboration with two British mathematicians, R. L. Brown and M. W. Thring, he published in the

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Proceedings of the Royal Society for July, 1949, an article in which he showed that the mathematical foundations of theoretical physics could be simplified and generalized by the assumption that the universe had five dimensions.[3]

Bennett calls the new axis the *eternity axis*, and it runs perpendicular to Minkowski's time axis; since it is also perpendicular to the three space axes, it defines a realm beyond the four-dimensional space-time world of physics. We may call this recently discovered world the "Eternity Domain of the Universe." Bennett tells us that the reason why we have no immediate experience of events which take place in this nonphysical realm is that we are "eternity blind."

Certain properties of the eternity domain can be deduced from the fact that it lies beyond both space and time in their physical sense. No physical measurements can be made in that world, because measurements are made with material measuring rods and clocks, and matter has no place in a nonphysical world. Since there can be no units of length or of time in the eternity domain, it must be described as *nonmetrical.* Separations in space cannot be defined, although there may be some kind of "mental separation." Long and short time intervals cannot be distinguished from one another, so that *duration* in time is meaningless. On the other hand, there are reasons to believe that a *sequence* in time has a well-defined meaning (see below). We may expect that there is a causal relationship between events in the physical and the nonphysical worlds.

We shall now make a fundamental assumption. In the nonphysical world beyond space and time, which has here been called the eternity domain, lies the ultimate origin of all things: energy, matter, life, consciousness, and mind. In short, all the characteristics of the world, physical as well as mental, are supposed to have their origin and their sustaining "roots" in this recently discovered extra-physical domain. This assumption should be regarded as a "working hypothesis," and its justification depends on whether or not it can explain, or rather help us to visualize, the relationship between physical and mental phenomena. In the following pages are given some examples of how the introduction of the eternity domain can help us to picture the nature of this relationship.

1. THE LOCATION OF MEMORIES

The following questions have often been asked: How is it possible for us to remember events which happened long ago? Where and in what form are our memories stored, and how are they brought forth in our conscious minds? It has sometimes been suggested that our memories are recorded in some form in the *material* brain, and that they are evoked by the association of ideas, by other memories, by visual observations, or, as recently found, by direct electrical stimulation of certain nerve cells in the cortex of the brain. But the brain is a *fluid* mass and must therefore, except at extremely low temperatures, be subject to the disruptive effects of the thermal motions of its molecules. Further, it is impossible for us to understand how the *material* structure in the brain can produce *mental* effects in our consciousness.

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[3] The story of the development of the theory of a five-dimensional universe, and how Bennett was influenced by the ideas of the Georgian mystic, Gurdjieff, is related in Bennett's book, "The Crisis in Human Affairs," London, Hodder and Stoughton, 1948. An extensive and technical description of the new ideas is presented in Bennett's more recent book, "The Dramatic Universe," London, Hodder and Stoughton, 1956. See also Maurice Nicoll's "Living Time," published by Vincent Stuart, London, 1952.

The introduction of the eternity domain has given us a clue to the solution of this difficult problem. We may think of the brain, as revealed by our sense organs and by our scientific instruments, as being a "projection" onto the space-time world of its inherently nonphysical counterpart in the eternity domain. We must always remember that, in whatever way our knowledge of the physical brain has been acquired this knowledge is in itself an *idea* of something which we can neither understand nor adequately describe. What we see with our eyes can only be a *partial* aspect of an underlying reality.

If a man's memory complex belongs to the immaterial eternity domain it can be expected to be free of any deterioration. This can explain the well-known fact that memories from our childhood can persist during a long life, in spite of the elimination of waste products, which goes on continuously during the metabolism in all living structures, so that we have a completely "new" brain after a few years.

The evocation of a particular memory element can now be pictured in the following way. A group of nerve cells in the brain is "stimulated" or "activated" by some means, for instance by being supplied with an extra amount of oxygen-carrying blood. The energy released in the oxidation or burning of the nerve tissue and the related modification of its electronic structure are associated with a corresponding change in the "spiritual" memory complex of the individual in the eternity domain (3, 4). The potential connections between the particular parts of the material brain and their spiritual counterparts then become effective, and the memory of a particular event appears in the consciousness of the owner of the brain. This idea is similar to Henri Bergson's theory of memories (5)—that our memories "are hidden from our knowledge by a veil of atoms." Because the physical brain is built of material elements having inertia, it takes energy and time to stimulate a memory element, but when it is evoked, it persists for a certain time. The process of remembering and of thinking is more or less "sluggish," so far as the brain mechanism is concerned, but in the nonphysical world these processes can be expected to be instantaneous.

The theory here expressed is supported by evidence offered by the so-called *panoramic death vision* (6). It has been reported for many centuries that a person near death, for instance by drowning, sometimes can see a panoramic view of his life, in some cases from childhood on. The moving pictures often show details which the person having the vision has forgotten long ago. It seems that when the oxygen supply to the brain has been cut off, and when the immaterial brain field is in the process of disentangling itself from its load of incorporated inert matter, normal mental sluggishness disappears. It is important to note that to an outsider the vision seems to last only a very short time, while to the participant, who later tells the story of his vision, it may have appeared to represent a long time interval, when a great number of things happened. Such death visions may be much more frequent than we think, since reports of such visions are obtained only from those who are resuscitated, and not from those who perish immediately following such visions. It is highly significant that the pictures seen are quite clear and do not overlap, as in a well-tuned television instrument. It indicates that in the eternity domain a *sequence* in time is meaningful, but a *duration* is not.

2. THE ORIGIN OF MATTER AND ENERGY

It is generally admitted that present-day science cannot give us any information about the *ultimate* origin of the matter and energy in the world of today. For instance, Hoyle's theory of a *continuous creation* (7) and the theory of a *cosmic explosion* developed by Alpher and Herman (8) give us no hint about the ultimate origin of energy. The introduction of an eternity domain in the universe makes it possible for us to form a mental picture of the origin of energy, and consequently of matter.

Let us think of the eternity domain as a continuum everywhere in contact with the four-dimensional space-time world, somewhat like the atmosphere in contact with the surface of the earth. At certain contact points, which may be called *energy sources* (9), energy *emerges* from the eternity domain into the space-time world. This energy may be thought of as previously existing in an undifferentiated and potential form, which can best be described as *mental*. At the very moment of their emergence, the elements of energy probably appeared in an extremely concentrated form, possibly as neutrons, as in Alpher's theory. Before their emergence, their location and size were undefinable, because these concepts have no meaning in the eternity domain, which lies beyond physical space. In the theory of continuous creation, the emergence goes on at all times, while in the explosion theory it took place at a particular time and with extreme violence. This explosion, if it actually occurred, represented the "birth" of the physical world and the beginning of physical or clock time. Recent studies indicate that this beginning of time took place between four and five billions of years ago (10).

3. THE ORIGIN OF LIFE AND OF CONSCIOUSNESS

Most biologists maintain that organic life on the earth is the result of a slow development which started with the formation of certain chemical compounds in the fluid water at the surface of the earth, the needed energy presumably being supplied by sunlight. From these simple beginnings, living cells were formed in some unexplained way and from them came higher forms of life, such as plants and animals. In the struggle for survival, a *natural selection* of living forms took place which favored the strong and hardy individuals, while the weaker and unfit forms were eliminated. This development was made possible by the close similarity between parents and offspring, coupled with seemingly accidental changes in their gene systems, the carriers of the hereditary factors.

In 1935 Burr and Northrop of Yale University proposed the "electrodynamic theory of life" (**11**). This led to an extensive investigation by Burr and his associates at the Section of Neuro-Anatomy of the Medical School of Yale University of the fine structure of the electric fields in which all living plants and animals are known to be imbedded. Many important facts were discovered during this investigation (**12**, **13**). It was found that the electric pattern had a complex structure which extended well beyond the visual limits of the living organism. If the oxygen supply necessary to maintain the metabolism was reduced, the field contracted, without change in structure, and disappeared completely at the death of the organism. Burr stated (**13**) that "it is hard to escape the conclusion that the electrical pattern is primary, and, in some measure at least, determines the morphological pattern." It seems that during growth fluid matter in the form of nourishment of particular electric structure (for example, proteins) is incorporated in a pre-existing electric field of complex structure and that it becomes directly observable by this form of "materialization."

Observations of this sort led the author in 1945 to propose the theory of "autonomous, living fields" (**14**), based on the assumption that the sources of the organizing electric fields cannot be located in the electrically charged particles of which matter is generally supposed to be built up, but should be regarded as *singularities* in a pre-existing force field with *topological* or *morphological*, but with no *metrical*, properties, such as size. Fields of this sort can be imagined as existing in an extremely contracted and dormant form in an egg cell or seed. Going one step further it may be postulated that the living field, when it is in its potential form, has no size at all, in which case it must be regarded as a *nonphysical potentiality*.[4]

The idea of the existence of "living sources" as the points of entrance of living "elements" emerging from the nonphysical eternity domain is in full harmony with the theory of "emergent energy" (9). Life seems to have emerged from "another world" than that described in the science of physics. The idea is almost identical with that presented by Northrop and Burr, in which the organizing fields are "primary properties of the universe, and that they are, to a greater or less extent, modified by the presence of matter, the field and the particles being thus dependently related." (Abstract of a letter from Burr to the author.)

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[4] Bennett, who regards this as a sixth dimension in the universe, calls it *Hyparxis*, which means "ableness to become."

The assumption that life originates in the eternity domain can help us to understand two important problems: the seemingly purposive activities in living organisms, and the ultimate origin of consciousness and mind. These two problems are intimately related, I believe.

The theory that the growth and development of a living organism is directed by a *purpose* is called *teleology*, which can be translated as "looking ahead"; it is particularly evident during embryonic development. Teleology can be regarded as the opposite to mechanism. With the general acceptance of mechanistic ideas, teleology has generally been regarded with much skepticism and often with hostility by most biologists, since it may open the door to mysticism of one sort or another. But some prominent biologists have accepted some form of teleology, for example, E. G. Conklin (15), who says "There may be good grounds for holding that mechanism and teleology are complementary views of nature, neither excluding the other." The problem of randomness *versus* directiveness has been discussed in great detail by Ralph S. Lillie (16) and similar ideas have been presented by E. W. Sinnott (17).

There is no reason to believe that an aggregation of atoms and molecules, simply because of its chemical structure, however complex this structure may be, can acquire any form of consciousness, not even of the simplest form. As an example, take the ability of men and many animals to perceive light and colors. It is known that this ability is a consequence of the existence of sensitive nerve cells in the retina of the eyes. From these nerve cells run nerve fibers to special neurones in the brain. When the nerve cells are stimulated by one means or another, be it by electromagnetic radiation, popularly known as "light," by a hard blow on the eyeball, by chemical irritants, or by static electricity applied directly to the optic nerve cells in the brain, there results a sensation of light or colors. How can the causal relationship between the *mechanical* stimulation and the subsequent *mental* sensation of light be explained? The hitherto unsolved problem of the relationship between mind and matter appears here in all its poignancy.

The introduction of the eternity domain, in which consciousness has its origin and its sustaining "roots," may help clarify the nature of this relationship.

A particular type of "living elements," which may be termed "potential optic nerve cells," have emerged from the mental eternity domain into the physical space-time world. After they became associated with an immaterial nerve system, and after the corresponding living fields had expanded and become materialized by the incorporation of matter of proper structure, they became parts of the gene system in the egg cell of the animal. (The fact that color blindness is linked to sex, indicates that certain "color genes" are located in the X-chromosomes.) When fully developed, the optic nerve cells become located in the retina of the eyes of the animal and are connected with complementary nerve cells in the occipital part of the brain. The connection between the optic nerve cells and their "cosmic" or "spiritual" counterparts in the eternity domain had never actually been lost, and when the nerve cells were stimulated by one means or another, which involves an absorption of energy, a "contact" between the physical nerve cell and its spiritual origin was established. As long as the field was in its activated and therefore sensitive state, a pre-existing potential association between the physical and the nonphysical world had become an actuality, and the animal or the man perceived a sensation of light or colors. Since there is reason to believe that the eternity domain forms an *integrated* and *nondivisible* whole, it may be said that a particular color sensation has its origin and its "roots" in the *World Soul*,[5] which itself is spiritual and purely mental. (This explanation of the origin of color sensations was first presented in the author's book, "The Soul of the Universe" (**18**).)

4. THE IMMORTALITY OF THE HUMAN SOUL

Peoples of many races and religions have expressed belief in the immortality of the human soul. The origin of this idea may lie in the hope of again seeing departed friends and relatives; it may have been fostered by assertions by wise or "holy" men; or it might have been due to a subconscious knowledge of the existence of another world than that experienced by our sense organs. It has been reported from many sources that messages have been received from the "spirit world" which contain information known to no one but the departed person, the information being later verified by an investigation. Natural science has reacted unfavorably to such ideas, largely because present-day science can give no explanation of such phenomena.

The introduction of an extra-physical dimension in our world picture has made it possible to form a scientific picture of the survival of the human soul at death. When the immaterial force field in the brain contracts, as do other living fields in the absence of oxygen, the contraction proceeds so that this field finally has no size at all, although its essential properties remain intact. When it has completely disappeared from the physical world, it becomes what may be described as a "living source." In other words, *it returns to the mental world from which it originally came.* As evidenced from the "death visions" described earlier, all memories in their minutest details can be recalled in an instant. This indestructible memory complex forms an essential part of the indivisible entity we call a *soul*, which apparently does not have to be associated with matter. It seems that the eternity domain is our real "home," and from there we make short excursions to the physical world of space and time. During such a trip or trips we learn many lessons which may be helpful in our further development.

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[5] The idea of a *World Soul* will be explained later.

5. THE MECHANISM OF MENTAL TELEPATHY

Thanks to systematic investigations, mainly carried on by Societies for Psychical Research in several countries and by the Psychology Departments in some Universities, particularly at Duke University, some facts have been established which make it necessary to postulate the existence of an extra-physical dimension, or rather a domain where physical separation in space seemingly has no effect. Experiments have shown that "thought pictures" can be transmitted over extremely long distances, and that the intensity and clearness of the mental impressions do not depend upon the distance between the sender and the recipient of the mental message (**19**). If a message were broadcast in all directions, and if it, while in transit, could be regarded as a particular form of energy, which could be described as "thought waves," its intensity would be expected to decrease with the square of the distance from the transmitting person. Also, due to the interference of other thought waves, the reception should be blurred. The specific fact, verified by many hundreds of testified observations, that the image of a dying person can appear clearly in the mind of another person indicates that, under unusual conditions, a mental contact between two persons widely separated in space can be established. No evidence of any time-lag has been established. Such conditions lead to the conclusion that the message or the mental picture does not "travel" in physical space as a form of energy, but in the extra-physical domain, where physical separation has no meaning.[6] Consequently, it is logical to believe that thoughts and mental pictures are transmitted *as such*, that is, in their original mental form. This might have been expected, since the eternity domain is itself a mental world.

6. THE ETERNITY DOMAIN AS A LIVING BEING

It is obvious that the mental characteristics attributed to the eternity domain are those manifested in the human mind. The human mind is not simply a collection of mental elements, like sensations, feelings, thoughts, and memories, but it is a highly *integrated unit*, to which may be given the name ***Person***, a word which literally means the sound emanating from a masked individual on a theatrical stage. A person is generally regarded as a being with some degree of intelligence, and the

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[6] The problem of long-distance telepathy has been thoroughly investigated by Sir Hubert Wilkins and Harold Sherman (**19**).

"mask" mentioned above may be considered something which hides him from the eyes of the public. This integrated mental unit called a person has often been called a *soul*, or its equivalent in the various languages spoken on the earth.

It has long been customary to speak about man as a combination of a soul and a body, *both* of which are highly integrated units. The integrated character of the human body is evidenced by the co-ordinated activities in our bodies, activities which reveal man not as a colony or aggregation of cells, but as a highly organized system in which the various parts perform specific functions necessary for the life of the organism as a whole.

The eternity domain is regarded as the realm in which all mental characteristics are "rooted." It is very difficult to picture this domain as a loose combination of mental elements, like various types of sensations, feelings of love and hate, pleasure and pain, of satisfaction and remorse. A characteristic property of the eternity domain is the absence of space separation, and therefore it should not be thought of as having any separated "parts." A separation in time may exist which separates the past from the future, although there is some evidence that even this kind of separation may be bridged under conditions which would make pre-cognition possible. The many striking phenomena of teleological guidance in the living world, as opposed to the unguided phenomena in the nonliving world, may be thought of as evidence of a *foresight* and *wisdom* far surpassing our own, but reflected in a small measure in the intelligence and purposive activities of the human race. Among men, the purpose of an action is often consciously known, while among animals purposive activities are often governed by *instincts*, which represent behavior patterns acquired through the experiences of a great number of generations and presumably preserved in some form and duplicated in the germ plasm of the race.

In view of what has here been said about the meaning of the term *person*, it is justifiable to apply this name to the nonphysical and mental realm called the eternity domain. It can now be described as an *Almighty*, *Wise* and *Living Person*, the *Creator* of all things, physical, mental, and spiritual. In our mind we have an "image" of this Person, and an idea of His existence and nature. We are created in His image, but although this image is incomplete and our idea of it distorted and colored by our own animal instincts, it is still an image of something greater than ourselves and of His whole creation. We can also understand that this Person in His wisdom may select one or more souls to carry important messages and admonitions to other human souls, while they are still associated with animal bodies and therefore have the animal instincts of selfishness and lust for unrestricted power. Some of these messages we can also hear directly when we listen to the voice of the "Cosmic Conscience." They tell us unequivocally that the essence of Divine law can be expressed in the simple admonition: "Love ye one another!"

For many centuries men of nearly all races have conceived of a Being or Spirit greater than themselves from which all things have originally come (**20**). It has been given various names, such as the Great Spirit, Jehovah, and Allah. Plato called it the *World Soul* (*Anima Mundi*), and Giordano Bruno gave his life in defending Plato's idea. In Vedantic idealism it has been called *Parama-Brahman*, a Sanskrit word meaning "The ultimate spiritual essence" (**21**).

In the English language it is called God.

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NATIONAL BUREAU OF STANDARDS NEWS

A STANDARD FOR MEASURING THE pH OF BLOOD

To increase the accuracy of pH measurements in the physiologically important pH range 7 to 8, the National Bureau of Standards is recommending a pH standard[1] for determining the pH of blood. A phosphate buffer solution, the proposed standard has a pH of 7.414 at 25 C. and 7.382 at 38 C. These standard pH values, accurate to within ±0.002 unit, will be useful to the research medical scientist, to the clinician, and to the physical scientist investigating acid-base behavior in the middle of the pH range.

To discover the relationships that exist among physiological function, pathological condition, and pH, medical and biological laboratories have long studied the acid-base relationships in blood and other physiological fluids. Because many of these fluids are well buffered, the detection of any systematic variation of acid-base balance with physiological condition would be expected to require precise pH measuring equipment. Such equipment is commercially available in the form of the newer highly-sensitive pH meters with glass electrodes. However, these commercial instruments measure only a difference in pH and will yield useful pH data only if a suitable reference standard is available. To meet this need, V. E. Bower, M. Paabo, and R. G. Bates of the Bureau's solution-chemistry laboratory developed the proposed standard. Their work results from a continuing program[2] to extend defined pH values through the pH scale and over a wide temperature range.

The new standard reference solution is prepared by dissolving 1.179 g (air weight) of potassium dihydrogen phosphate and 4.303 g (air weight) of disodium hydrogen phosphate in distilled water and diluting the solution to 1 liter at 25 C. The molalities of the two salts are, respectively, 0.008695 and 0.03043, and the ionic strength is 0.1.

Standard pH values—which were derived from emf measurements—have been assigned to the reference phosphate solution at intervals of 5 degrees from 0 to 50 C. To avoid the theoretical and practical difficulties inherent in the estimation of liquid junction potentials, emf measurements of cells without liquid junction were used.

An arbitrary, nonthermodynamic convention concerning single-ion activity must be adopted when defining pH. Because no one definition has been widely accepted, useful comparisons of highly precise pH data from different sources have been impossible until now. However, Bates and Guggenheim have suggested a simple, useful way[3] of defining the activity coefficient of the chloride ion, which is needed to assign standard pH values. This definition has recently been endorsed by standardizing groups in the United States and Great Britian. According to this proposal, the single-ion activity coefficient (γ_{Cl}) of the chloride ion in dilute solution is so defined that it closely approximates the mean ionic activity coefficient of simple uni-univalent strong electrolytes in aqueous solution up to ionic strength (I) of about 0.1. The exact relationship adopted is:

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[1] For further technical information, see "A Standard for the Measurement of the pH of Blood, and Other Physiological Media," V. E. Bower, M. Paabo, and R. G. Bates, *J. Research NBS*, Vol. 65A (Phys. and Chem.), p. 267 (May–June 1961).

[2] "Standardization of the pH Scale," *NBS Tech. News. Bull.*, Vol. 31, p. 138 (1947).

$$-\log\gamma_{\text{Cl}} = \frac{A\sqrt{I}}{1 + 1.5\sqrt{I}}$$

where A is a constant of the Debye-Hückel theory, the value of which depends on the temperature.

With this new convention, standard pH values accurate to within ±0.001 unit can be assigned to selected reference solutions with pH between 2 and 12. The ionic strength of these solutions should not exceed 0.1.

The standard pH scale established by the Bureau several years ago is defined in terms of four fixed points which were obtained with standard reference solutions. These primary standards[4] were found to have a satisfactory degree of internal consistency.[5] The values assigned to the new pH standard for blood are also consistent with these four standards. Together, these standards define a pH scale which, though conventional, is practical, convenient, and useful.

FAST ELECTRON DOSE FROM A NARROW BEAM SOURCE

The National Bureau of Standards has made a calculation of the penetration in air of fast electrons from a beam source. The work[6] was sponsored by the Atomic Energy Commission and was part of a program to develop methods for calculating the dose from a source geometry involving two spatial variables. Possible applications of such calculations include the determination of the spatial distribution of the dose in therapy involving electron beams and the estimation of the resolving power of large single crystals in high-energy radiography.

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[3] R. G. BATES AND E. A. GUGGENHEIM, *Pure and Applied Chemistry*, Vol. 1, p. 163 (1960).

[4] The materials from which the standard solutions are prepared are described in NBS Circular 552, Standard Materials. This Circular may be ordered for 35 cents from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.

[5] R. G. BATES, *Chimia*, Vol. 14, p. 111 (1960).

[6] For further information, see "Calculated Energy Dissipation Distribution in Air by Fast Electrons from a Gun Source," by JOHN E. CREW, *J. Research NBS*, Vol. 65A (Phys. & Chem.), No. 2, p. 113 (1961).

The experimental situation is that of a collimated beam of monoenergetic electrons originating at a point in the medium. As the electrons move about, their energy is dissipated. The basic quantity to be determined is the energy dissipated in a small volume located at specified distances from the source point and the center line of the initial beam.

Calculations of the energy dissipation distributions for fast electrons in infinite and homogeneous media have been reported for the simplest source geometries.[7,8] The present calculation represents an extension of the earlier work and is a special application of the moment-fitting techniques developed at the Bureau and applied initially to problems of X-ray penetration.[9,10]

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BOOK REVIEWS

RADIO WAVES IN THE IONOSPHERE, by K. G. Budden. 509 pages, 6 × 9 in. New York, Cambridge University Press, 1961. Price, $18.50.

The enormous success and expansion during the last twenty years of such fields as nuclear, quantum and solid state physics have kept many other important branches of science out of the limelight. The physics of the ionosphere is one of the latter and, besides the electrical engineers, many physicists have maintained a great deal of interest and activity in this area. The public and international interest in satellites and space travel has contributed to stimulate a new departure in this and other "space sciences."

At this juncture, Ratcliffe's monograph on "The Magneto-Ionic Theory" (Cambridge, 1959) and the present book provide an up-to-date and comprehensive survey of this field of physics. While Ratcliffe's works emphasize the physical aspects, the present treatise gives the full mathematical treatment of the problems involved, including those formulated in Ratcliffe's.

In the first half of the book the subject is introduced starting from the basic Maxwell and constitutive equations, and the propagation of electromagnetic waves in homogeneous (isotropic and anisotropic) media. This first half deals with the phenomena where the "ray-approximation" is valid. The second half gives the "full wave theory." The subject matter is developed carefully, all definitions clearly stated and analyzed, standard definitions (for example, reflection and transmission coefficients) revised. The book is a mine of very diverse mathematical techniques (some, like the W.K.B. method, widely used in other fields) well exposed and adapted to the situation at hand.

The research worker and the student alike will appreciate the methodical presentation, the careful bibliography, and the diversity of problems solved or proposed.

HENRI AMAR
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HANDBUCH DER PHYSIK (ENCYCLOPEDIA OF PHYSICS), VOL. X, STRUCTURE OF LIQUIDS, edited by S. Flügge. 320 pages, diagrams, 6¾ × 9¾ in. Berlin-Wilmersdorf, Springer-Verlag, 1960. Price, DM 76.80.

While the physics and chemistry of solids have been experiencing an extraordinary growth and popularity, the investigations of the liquid state have been relatively limited in number and sometimes in scope. This is due partly to the fewer technological applications of liquids and partly to the inherent and formidable difficulties of the subject. Nevertheless a number of devoted and talented theoreticians and experimentalists have been pursuing an active research in this field. The success achieved has been substantial if not spectacular.

It is the purpose of this handbook volume to give a methodical survey of the present theories of the Structure of Liquids. This survey is divided into three parts. In Part 1, H. S. Green (well known for his early work with Max Born and for his book on the molecular theory of liquids) first analyzes the general nature of the liquid state, and the quantitative approach of the distribution function. He then exposes the modern views on the structure of uniform, non-uniform and "quantum" liquids. In Part II, Professor Syn Ono (Radiation Laboratory, University of Tokyo) surveys the molecular theory of surface tension in liquids. He describes clearly the three present approaches to this subject, namely, (*a*) through statistical mechanics, (*b*) through thermodynamics and "quasi-thermodynamics," (*c*) through lattice theories. The third and shortest part, by F. P. Buff (University of Rochester), is devoted to the Theory of Capillarity. Gibbs' thermodynamic formulation is stressed and several applications are given.

The nature of the subject makes it necessarily highly mathematical. The exposition is, on the whole, methodical and thorough, and the bibliography is excellent. The German-English dictionary appendix is hardly necessary since the three articles are written in English. This appendix could have been advantageously replaced by a comprehensive and descriptive synthesis facilitating for the reader the long journey through the long mathematical treatments.

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Temple University

THE FERMI SURFACE, edited by W. A. Harrison and M. B. Webb. 356 pages, diagrams, 8½ × 11 in. John Wiley & Sons, Inc., New York, 1961. Price, $10.00.

This is the proceedings of an International Conference on the Fermi surface which was held at Cooperstown, N. Y., during August 22–24, 1960. The conference was well attended and the papers and discussions present a fair picture of the most recent thinking with regard to the electronic structure of metals and alloys. Both theory and experiment are well represented in the proceedings. It is satisfying to find large and growing areas of agreement between the two. This is in marked contrast to the picture ten to fifteen years ago when it seemed that attempts to improve on the simple free electron theory of metals ended in poorer agreement with experiment. Today, theoretical methods of considerable power have given us some insight into why the free electron model is as good as it is. Another striking change is the abandonment of analytical forms for the shape of the Fermi surface and their replacement by geometrical and topological models. The major experimental techniques—the de Haas-van Alphen effects, galvanomagnetic effects, cyclotron resonance, anomalous skin effect, magnetoacoustic effect, and transport effects—are described in separate sessions of from three to six papers each. Finally, the results of the conference are summarized by M. H. Cohen (theory) and A. B. Pippard (experiment). This is an excellent reference work whose usefulness is enhanced by the thoughtfulness of the editors in providing a nine page subject index.

F. J. DONAHOE
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FRAME ANALYSIS: A UNIFIED INTRODUCTION TO THE MATRIX ANALYSIS OF STRUCTURES, by A. S. Hall and R. W. Woodhead. 247 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1961. Price, $8.50.

The electronic computer makes possible the analysis of complex framed structures by matrix methods. The authors of this concise yet comprehensive book successfully demonstrate that the representation in matrix form of a number of familiar methods of structural analysis leads to a more unified approach and helps to promote clearer understanding of the individual techniques. However, only an elementary knowledge of matrix methods is required, for which sufficient background is supplied in an Appendix.

The presentation is divided into three parts. The first two introductory chapters summarize the basic definitions and concepts, such as equilibrium, compatibility, etc., and distinguish as well as relate the two basic and reciprocal techniques for the analysis of indeterminate structures: Action methods (such as virtual work) in which restraints are replaced by redundants of the proper magnitude to restore continuity; and displacement methods (such as slope deflection) which solve for the magnitude of the displacement required to restore equilibrium. Each of the two main parts of the book is based on one of these approaches. Their duality is demonstrated by solution of identical problems by both methods.

The first two chapters of Part I discuss the calculation of influence and flexibility coefficients and illustrate their use by a number of fully worked representative examples. Chapter 5 demonstrates the advantages of transforming the flexibility coefficients so as to produce more reliable numerical solution by change of scale factors, and of redundants; and Chapter 6 numerizes the mathematical technique for axis transformations. The final two chapters treat the flexibility analysis of a single member, and apply the techniques of transformation of restraints and axes. The use of the elastic center, and the column analogy, are shown to be particular cases of the general methods given.

Part II, on Displacement Methods, is introduced by a discussion of the slope deflection method. The two succeeding chapters derive matrices for the stiffness of a single member and of a frame, and include a brief treatment of the effects of incomplete fixity. Chapter 12 reviews two relaxation methods related to slope deflection as applied to plane frames: Moment distribution, and "group relaxation," which has been used where the presence of sidesway complicates moment distribution. A final chapter deals with the method of coefficients of restraint, a form of moment distribution employing modified stiffness coefficients and carry-over factors which obviate the need for iteration.

This book, written for the senior student and practicing engineer, is concerned with the application of mathematical method to structural analysis, and is not intended as a treatise on matrix calculus. The authors are to be highly commended for producing a very useful and instructive work within the limits which they have set themselves, and which they invite the reader to extend by further study of the references they supply.

K. L. CAPPEL
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ADVANCES IN DOCUMENTATION AND LIBRARY SCIENCE, VOL. III, PART 1, INFORMATION RETRIEVAL AND MACHINE TRANSLATION, edited by Allen Kent. 686 pages, illustrations, 6 × 9 in. New York, Interscience Publishers, 1960. Price, $23.00.

An estimated 70,000 scientific journals appearing in many foreign languages will be published this year. Significant research results reported in these publications will make vital contributions in every area of scientific and technical endeavors. One great problem that presently exists is that a large percentage of this information may never reach those individuals who could make use of it. The magnitude of the world's printed scientific output coupled with the language barrier has created a communication problem that appears will only be solved by new methods and techniques. These methods and techniques are directed towards the use of computers for the retrieval of information and the translation of languages.

Western Reserve University's Center for Documentation and Communication Research has been one of the pioneering groups that has carried out research in this area. A series of volumes concerned with the many facets of this subject have been published over the past years. The present volume under review is based on papers presented at the International Conference for Standards on a Common Language for Machine Searching and Translation, sponsored by Western Reserve University and Rand Development Corporation held in Cleveland, Ohio, September 6–12, 1959. The 21 papers were presented by specialists drawn from all over the world, who were concerned with some methods of "working toward a common machine language or a series of compatible machine languages for processing scientific and technical literature so that it may be searched, selected, correlated, and translated by automatic equipment." The papers deal with many areas: 1. Classification and coding of specific types of literature for certain user groups, 2. Description of the specific computers and computer-like devices, 3. The problems of analysis of scientific information for the purpose of storage and retrieval and 4. Description of specific systems that are now operational.

This volume represents a significant review of the state of the art as it is now practiced in organizations located throughout the world. The principal criticism is that the papers reflect a certain chaos that still exists in the information retrieval field. As Dr. Ranganathan states in the appended discussion in this volume, the terminology varies from paper to paper with inconsistencies and lack of general standards. The volume will not be read from cover to cover by most readers but will stand as an important reference work. A more prudent editorial policy might have pared the size of the volume down to manageable proportions and also deleted some of the over-lapping and duplication. Adequate author abstracts would have been of considerable help in digesting some of the material.

JOHN P. McGOWAN
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BOOK NOTES

Physical Chemistry of Process Metallurgy (in two parts), edited by George R. St. Pierre. 1374 pages, illustrations, 6 × 9 in. New York, Interscience Publishers Inc., 1961. Price: $22.50 (Part 1); $25.00 (Part 2).

These two volumes (Vols. 7 and 8 of the Metallurgical Society Conference series) are the proceedings of an international symposium sponsored by the AIME in 1959. Part 1 deals with the physical chemistry of metallurgical and oxide phases, thermodynamics of metals, transport and mixing, the nature and structure of liquid metals, and solubility and phase equilibria in metals systems.

Part 2 covers process reaction rates and mechanisms, solidification of metals, properties of halide and sulfide melts, industrial applications of principles, desulfurization in steel plants, and process control and statistical methods.

Discussions are included with a majority of the individual papers.

Polymeric Materials, by Charles C. Winding and Gordon D. Hiatt. 406 pages, diagrams, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $12.00.

The purpose of this book is to introduce the reader to polymeric materials so that he may be able to understand them and cooperate with others actively working in the fields of plastics, rubber, protective coatings and textile fibers. The book can also be used as a text for an introductory college course, but a good knowledge of chemistry is a prerequisite. It is not intended to aid the specialist in any of the polymeric fields covered.

Engineered Castings, by Glenn J. Cook. 240 pages, diagrams, illustrations, plates, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $8.50.

A practical approach to all phases of engineered castings is used in this new book aimed at design engineers, draftsmen, materials engineers, purchasing agents, and cost-reduction specialists, as well as at foundrymen, patternmakers, foundry engineers and managers. It covers how castings are designed, made and used, and gives practical suggestions for use in selecting cast material specifications and in buying castings. The author, Manager of Marketing in GE's Foundry Department, expresses the hope that the book will stimulate young men to find careers in the foundry industry.

Heat Transfer, by Benjamin Gebhart. 454 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $10.75.

This book describes the more important physical processes, theories, and methods of analysis for the heat-transfer field. Although it is designed as a text, it will be of use to the practicing engineer as well as the student. Basic theories and formulations of conduction, radiation and convection are presented, and the mathematical statement of problems is emphasized. The author's intent is to demonstrate the diverse and special considerations whereby general formulations are adapted to particular applications. Laplace transforms are not used in the solution of partial differential equations. For use as a text in a first course, the author suggests omission of much of Chapter 6 on fluid flow and all of Chapter 13 on experimental analogues. In some cases, the same symbol denotes several different quantities, which may prove a little confusing.

The Physical Theory of Transistors, by Leopoldo B. Valdes. 358 pages, diagrams, 6½ × 9¼ in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $10.50.

This book is intended for a graduate course in transistor physics, and as a reference for those already working in this expanding field. In order to unify the various concepts making up the study of transistors, some introductory material is included.

The text analyzes the flow of current through semiconductor materials and derives relationships between the electrical characteristics of transistors and their physical structure. The book is divided into three parts, the first giving introductory or review material. The second part is devoted to the more physical aspects of transistor theory and deals in generalities, and the third part analyzes specific device structures and the electrical characteristics of diodes and transistors.

SPECIAL CERAMICS, edited by P. Popper. 363 pages, diagrams, illustrations, 5½ × 8½ in. New York, Academic Press, Inc., 1961. Price, $10.50.

This book represents the proceedings of a Symposium held by the British Ceramic Research Association in the summer of 1959. The symposium was held to give an opportunity for information to be exchanged concerning the research on non-oxide ceramics conducted by the BCRA in cooperation with various government departments and industrial firms, in response to the growing demand for new high temperature materials. Topics dealt with at the Symposium included: Properties and Structure, Measurements; Preparation and Properties of Nitrides; Preparation and Properties of Other Non-Oxides; Furnaces, Techniques, Analyses, etc.

THE ENCYCLOPEDIA OF THE BIOLOGICAL SCIENCES, edited by Peter Gray. 1119 pages, diagrams, illustrations, 7 × 10 in. New York, Reinhold Publishing Corp., 1961. Price, $20.00.

The biological sciences (from Abiogenesis to Zoogeography) are covered in over 800 articles specially written for this new encyclopedia. Its up-to-dateness is indicated by the fact that it includes articles on biological warfare (by LeRoy Fothergill) and on space biology (by O. E. Reynolds and John M. Talbot). It deals with developmental, ecological, functional, genetic, structural and taxonometric aspects of the biological sciences, and covers several topics in biophysics and biochemistry. The contributors include experts from many countries. References are given at the end of most of the articles. The expert and the layman alike will find answers to their questions in this newest of the specialized encyclopedias.

POWDER METALLURGY, edited by Werner Leszynski. 843 pages, diagrams, illustrations, 6 × 9 in. New York, Interscience Publishers, 1961. Price, $25.00.

The papers in these proceedings were presented at the International Powder Metallurgy Conference in June, 1960. Four main divisions cover: Mechanism of Sintering (13 papers); Dispersion Strengthening (6 papers); Technology and Methods (12 papers); and Materials and Applications (12 papers). Discussions are included with 29 of the papers. Paul Schwarzkopf (one of the pioneers in this field) gave the keynote address, which is included as an introduction to the volume. It covers the last fifty years of the powder metallurgy art. The papers are about equally divided between theoretical and practical approaches, and deal with powder metallurgy as a science, a technology, and an industry. The stature of the contributors and the wide range of material covered make this a fine and lasting contribution to the literature in this important field.

QUANTUM MECHANICS, by Eugen Merzbacher. 544 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1961. Price, $12.00.

With emphasis on contemporary developments, this book represents a new and valuable contribution to the subject of non-relativistic quantum mechanics. The development of the discussion of quantum mechanics is presented in three stages by the author. In the first, the ordinary wave mechanics is developed, enabling the student to handle actual problems of atomic and nuclear physics as early as possible. The matrix form of quantum mechanics is explicitly introduced as a useful tool in formulating one dimensional problems and is used again in the treatment of spin, in the second stage. In the third stage, the two versions of quantum mechanics are unified in the general and more abstract Dirac formulation which the author employs throughout the second half of the book

FUNDAMENTALS OF MODERN PHYSICS, by Robert M. Eisberg. 729 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1961. Price, $10.50.

This text provides a rapid but thorough treatment of the important developments leading up to Schroedinger quantum mechanics; a careful introduction to the basic theory of that subject, using an inductive rather than a postulatory approach; and illustrations of the theory through a complete discussion of a large number of typical calculations. Finally, he uses the theory to give a detailed explanation of the properties of atoms and nuclei. The author launches each topic near its beginning by means of arguments that presuppose only a background in elementary physics, and in mathematics through intermediate calculus.

ENGINEERING DATA FOR PRODUCT DESIGN, edited by Douglas C. Greenwood. 425 pages, diagrams, 8½ × 11¼ in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $10.00.

This manual has been prepared to help the engineer solve design problems. Through charts, formulas, tables, and special articles (taken from *Product Engineering*) the author places valuable and time-saving data at the fingertips of designers, about metals and alloys, nonmetallic materials, finishes, and design analysis. Clearly illustrated and carefully indexed, the manual should be of real help in answering questions about beams, torque, gears, shafts, control, and the application of electricity, electronics, hydraulics, and pneumatics to product engineering.

ELECTRICAL ENGINEERING, by S. B. Hammond. 547 pages, diagrams, 7 × 9½ in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $8.75.

This text is intended to be used for engineering students not majoring in electrical engineering. It covers the basic principles of electricity, its use and control. Sample chapter headings are: Characteristics of Voltages and Currents; Electric Networks; Nonlinear Resistor Circuits; Electron Physics; Feedback Circuits; Linear Approximations to Machine Analysis; Digital Computers; and Magnetic Fields and Circuits. The three main divisions deal with electric circuit analysis, electronics, and conversion of energy.

DESIGN AND CONSTRUCTION OF PORTS AND MARINE STRUCTURES, by Alonzo DeF. Quinn. 521 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $16.00.

In this work, the author brings together the essential concepts of designing and building modern shipping-terminal, harbor, and offshore marine structures—along with the working principles and practices that can be adapted to specific project requirements. He provides you with authoritative construction know-how and recommendations for achieving top efficiency and economy in meeting the demands of today's advanced engineering techniques. This is the only U. S. published book dealing solely with ports and marine structures.

FUNDAMENTALS OF CIRCUIT THEORY, by Norman Balabanian. 548 pages, diagrams, 6 × 9 in. Boston, Allyn and Bacon, Inc., 1961. Price, $9.50.

The author, Norman Balabanian of the Electrical Engineering Department of Syracuse University, states in his preface that this book is his attempt at giving an account of the concepts and techniques of circuit theory. The book's 13 chapters include Resistance Networks, Complete Response of Simple Networks, Sinusoidal Steady-State Response, The Complex-Frequency Variable, Two-Port Networks, Electronic Devices and Their Models, Operational Diagrams and Signal Flow Graphs, and Complete Response to Arbitrary Wave Forms.

Intended primarily as a textbook, the author has attempted to leave the student with the impression that circuit theory can still challenge his inventiveness, that it still has plenty of vitality and that it is not, by any means, a finished subject.

AN INTRODUCTION TO THE THEORY AND PRACTICE OF TRANSISTORS, by F. F. Roberts and J. R. Tillman. 335 pages, diagrams, plates, 5½ × 8½ in. New York, John Wiley & Sons, Inc., 1961. Price, $8.00.

The authors have prepared this book to present a wide treatment of the subject in one volume, for newcomers to the transistor and related semiconductor fields. Part 1, covering basic physical theory, will be interesting to electronics engineers and experimental physicists; Part 2 covers technology, properties and applications. Unipolar transistors are omitted, as are the merits of different types of sealing and the factors affecting reliability, although a brief section is devoted to the point-contact transistor.

MAGNETIC CONTROL OF INDUSTRIAL MOTORS, by Gerhart H. Heumann. Part 1: A-C CONTROL DEVICES AND ASSEMBLIES, 273 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1961. Price, $9.00. Part 2: A-C MOTOR CONTROLLERS, 325 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1961. Price, $9.00.

This three-volume work divides the subject of magnetic control of industrial motors logically into three parts, two of which are reviewed here. The third (on D-C Motor Controllers) was not available for comment at the time this issue of the JOURNAL went to press. Primarily an application book, the work covers magnetic devices and accessories used for control of industrial motors. It deals with principal circuits, selection of controller sizes and components, economic factors affecting controller selection, motor protection, and safety codes and standards.

Part 1 includes chapters on: Symbols and Diagrams; Alternating-Current Contractors and Relays; Accessories; Control-Circuit Devices; Resistors and Rheostats; Solenoids, Thrustor Mechanisms, and Brakes; Mechanical Design of Controllers; and Maintenance.

Part 2 covers: Drive Characteristics; Control of Squirrel-Cage Motors; Control of Wound-Rotor Motors; Logic Functions and Static Switching; Motors Protection; and Motor Branch-Circuit Protection.

INTRODUCTION TO COMPLEX ANALYSIS, by Zeev Nehari. 254 pages, diagrams, 6 × 9 in. Boston, Allyn & Bacon, Inc., 1961. Price, $7.50.

This text provides an introduction to the basic techniques and methods of complex analysis. Since these techniques may be applied in many areas, the book should be useful for many students in mathematics, engineering and the sciences. Suitable for a senior or first-year graduate course, the material presented covers complex variables, analytic functions, complex integration, the application of Cauchy's theorem, conformal mapping, physical applications such as boundary-value problems and two-dimensional fluid flow. Sections on analytic continuation, the symmetry principle and the harmonic conjugate may be omitted if it is desired to place emphasis on applications.

SEMICONDUCTOR DEVICES AND APPLICATIONS, by R. A. Greiner. 487 pages, diagrams, $6\frac{1}{4} \times 9\frac{1}{4}$ in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $12.50.

This new textbook, suitable for the senior or starting graduate student, emphasizes the proposition that an engineer must have a deeper understanding of the operation of electron devices than merely the ability to use specific devices in circuits. The book contains a thorough discussion of the properties of semiconductors and junctions as a preliminary to the discussion of transistor operation. Characteristics and parameters are derived from basic physical concepts, and extensive application of transistors to a variety of small and large signal circuits is included. The author has inserted many completely worked examples.

INTRODUCTORY SYSTEM ANALYSIS: SIGNALS AND SYSTEMS IN ELECTRICAL ENGINEERING, by William A. Lynch and John G. Truxal. 445 pages, diagrams, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $7.50.

The primary objective of this new text is to present the fundamental concepts and techniques of modern electrical engineering by introducing the viewpoints, approaches, and methods of the electrical engineer, and to familiarize the reader with electrical devices and equipment. It also provides the requisite foundation for future study, and presents a definitive picture of the role of the electrical engineer in the modern interdisciplinary systems engineering efforts, both military and industrial.

In addition to the basic techniques of system analysis being presented simply, but thoroughly, there are also several unusual topics treated, among them signal-flow graphs and analog computers.

PUBLICATIONS RECEIVED

RADIOACTIVE SUBSTANCES, by M. Curie. A translation from the French of the classical thesis presented to the Faculty of Sciences in Paris. 94 pages, 4½ × 7¼ in. New York, Philosophical Library, 1961. Price, $2.75.

THE WORLD OF GEOLOGY, edited by L. Don Leet and Florence Leet. 262 pages, illustrations, 5½ × 8½ in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $4.25.

A MIDWESTERN ALMANAC, by Austin L. Rand and Rheua M. Rand. 176 pages, diagrams, 6 × 9 in. New York, The Ronald Press Company, 1961. Price, $4.50.

TEXT-BOOK ON SPHERICAL ASTRONOMY, by W. M. Smart. Fourth edition, reprinted, 430 pages, diagrams, 5½ × 8½ in. New York, Cambridge University Press, 1961. Price, $3.95 (paper).

ELEMENTS OF MATHEMATICS, by J. Houston Banks. Second edition, 465 pages, diagrams, 6 × 9 in. Boston, Allyn and Bacon, Inc., 1961. Price, $6.95.

HUMAN HEREDITY, by Jean Rostand. Translated from the French by Wade Baskin. 135 pages, 4½ × 7¼ in. New York, Philosophical Library, 1961. Price, $4.75.

PHYSICAL CHEMISTRY, by E. A. Moelwyn-Hughes. Second revised edition, 1333 pages, plates, diagrams, 6 × 9½ in. London, Pergamon Press, 1961. Price, 84s. net.

MICHAEL FARADAY. A LIST OF HIS LECTURES AND PUBLISHED WRITINGS, compiled by Alan E. Jeffreys, with a foreword by Sir Lawrence Bragg. 86 pages, plates, 7 × 9¾ in. New York, Academic Press Inc., 1961. Price, $7.00.

ELECTRICAL ESTIMATING, by Ray Ashley. Third edition, 437 pages, plates, diagrams, 7¼ × 10 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $12.50.

ELEMENTARY FLUID MECHANICS, by John K. Vennard. Fourth edition, 570 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1961. Price, $7.95.

INTRODUCTION TO NUCLEAR ENGINEERING, by Raymond L. Murray. Second edition, 384 pages, illustrations, diagrams, 6 × 9 in. Englewood Cliffs, Prentice-Hall, Inc., 1961. Price, $12.00 (trade); $9.00 (classroom adoption).

INTRODUCTION TO HI-FI, by Clement Brown. 188 pages, diagrams, illustrations, 5½ × 8½ in. New York, Gernsback Library, Inc., 1961. Price, $3.20 (paper); $5.00 (cloth).

PROBLEMS IN QUANTUM MECHANICS, by I. I. Gol'dman, V. D. Krivchenkov, V. I. Kogan, and V. M. Galitskii, translated, edited and arranged by D. ter Haar. 394 pages, diagrams, 6 × 9 in. New York, Academic Press Inc., 1961. Price, $12.00.

TELEVISION AND RADIO REPAIRING, by John Markus. Second edition, revised and up-dated, 568 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $8.95.

CYBERNETICS, by Norbert Wiener. Second edition, revised, 212 pages, 6 × 9 in. New York, John Wiley & Sons, Inc., 1961. Price, $6.50.

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NEW BOOKS IN THE FRANKLIN INSTITUTE LIBRARY

PHYSICS

FIELD THEORY FOR ENGINEERS, by P. Moon. Van Nostrand, 1961.

COMPUTING METHODS AND THE PHASE PROBLEM IN X-RAY CRYSTAL ANALYSIS, edited by Ray Pepinsky. Pergamon, 1961.

THE ENCYCLOPEDIA OF MICROSCOPY, by G. L. Clark. Reinhold, 1961.

AN INTRODUCTION TO THE THEORY OF VIBRATING SYSTEMS, by W. G. Bickley. Clarendon, 1961.

MATERIALS FOR NUCLEAR ENGINEERS, edited by A. B. McIntosh. Interscience, 1960.

PHYSICS OF SEMICONDUCTORS, by A. F. Ioffe. Academic, 1960.

PHYSICS FOR THE INQUIRING MIND, by E. M. Rogers. Princeton University Press, 1960.

GENERAL RELATIVITY AND GRAVITATIONAL WAVES, by J. Weber. Interscience, 1961.

TECHNIQUES OF HIGH ENERGY PHYSICS, by D. M. Ritson. Interscience, 1961.

X-RAY MICROSCOPY, by V. E. Cosslett. Cambridge University Press, 1960.

THE ROTATION OF THE EARTH, by W. H. Munk. Cambridge University Press, 1960.

DISPERSAL RELATIONS AND CAUSAL DESCRIPTION, by J. Hilgevoord. North-Holland, 1960.

MAGNETO-FLUID DYNAMICS, edited by F. N. Frenkiel. National Research Council, 1960.

QUANTUM MECHANICS, by J. L. Powell. Addison-Wesley, 1961.

CHEMISTRY AND CHEMICAL ENGINEERING

CERAMICS, by P. W. Lee. Reinhold, 1961.

PORCELAIN ENAMELS, by A. I. Andrews. 2nd ed. Garrard Press, 1961.

AN INTRODUCTION TO TRANSITION-METAL CHEMISTRY: LIGAND-FIELD THEORY, by L. E. Orgel. Wiley, 1960.

INTRODUCTION TO PETROLEUM CHEMICALS, by H. Steiner. Pergamon, 1961.

PHYSICAL CHEMISTRY, by W. F. Sheehan. Allyn and Bacon, 1961.

CIVIL ENGINEERING

AMERICAN RAILROADS, by J. F. Stover. University of Chicago Press, 1961.

ADVANCED DESIGN IN STRUCTURAL STEEL, by J. E. Lothers. Prentice-Hall, 1960.

THE DESIGN OF CYLINDRICAL SHELL ROOFS, by J. E. Gibson. 2nd ed. Van Nostrand, 1961.

PAPERS ON BUILDING CONSTRUCTIONS. American Society for Testing Materials, 1960.

SYMPOSIUM ON TECHNICAL DEVELOPMENTS IN THE HANDLING AND UTILIZATION OF WATER AND INDUSTRIAL WASTE WATER. American Society for Testing Materials, 1960.

ELECTRICAL ENGINEERING

LINEAR GRAPHS AND ELECTRICAL NETWORKS, by S. Seshu. Addison-Wesley, 1961.

TRANSISTOR CIRCUIT ANALYSIS, by M. V. Joyce. Addison-Wesley, 1961.

ESSENTIALS OF DIELECTROMAGNETIC ENGINEERING, by H. M. Schlicke. Wiley, 1961.

ELECTRONICS, by P. M. Chirlian. McGraw-Hill, 1961.

SEQUENTIAL DECODING, by J. M. Wozencraft. M.I.T. Press, 1961.

ELSEVIER'S DICTIONARY OF AUTOMATION, COMPUTERS, CONTROL, AND MEASURING IN SIX LANGUAGES. Elsevier, 1961.

ADAPTIVE CONTROL SYSTEMS, edited by E. Mishkin. McGraw-Hill, 1961.

RADIO WAVES IN THE IONOSPHERE, by K. G. Budden. Cambridge University Press, 1961.

LECTURES ON COMMUNICATION SYSTEM THEORY, by E. J. Baghdady. McGraw-Hill, 1961.
TRANSMISSION OF INFORMATION, by R. M. Fano. M.I.T. Press, 1961.

METALLURGY

X-RAY METALLOGRAPHY, by A. Taylor. Wiley, 1961.
THEORY OF METAL CUTTING, by P. H. Black. McGraw-Hill, 1961.
RARE METALS HANDBOOK, by C. A. Hampel. 2nd ed. Reinhold, 1961.
TECHNOLOGY OF NON-METALLIC COATING, by A. Ya. Drinberg. Pergamon, 1960.
COLLOQUE DE MÉTALLURGIE: CORROSION, SÈCHE ET AQUEUSE. Centre d'études nucléaires de Saclay, 1960.
METALL UNTER DEM MIKROSKOP, von E. Kauczor. Springer, 1960.

CURRENT TOPICS

New Superconductor Material.—A new superconductor material which offers no resistance to electrical current has been developed by Atomics International, a division of North American Aviation, Inc., under a research contract with the Atomic Energy Commission.

A wire which can be easily coiled has been drawn from the new superconductor. At liquid helium temperatures (−452 degrees F.) the wire has conducted 100,000 amperes of electrical current per square centimeter in a moderately high magnetic field (30,000 gauss). The performance of this wire may be roughly equated with operation of a household electrical appliance at about 100,000 watts from a normal-size (110 volt) wall outlet.

The new material is a cold-worked alloy made up of approximately three parts of niobium to one part of zirconium. The alloy is malleable and strong and can be made into wires, bars, strips and other shapes without losing its superconducting properties. It is expected to retain these characteristics in substantially higher magnetic fields. Its development makes possible the fabrication of superconductor magnets to replace conventional bulky and relatively inefficient iron-core electromagnets. The superconductor magnet has no iron core and may produce magnetic fields many times higher than conventional electromagnets in common application today.

Potentially, the use of superconducting magnets of 100,000 gauss strength for high energy particle accelerators, or in connection with controlled thermonuclear research devices, could substantially reduce their cost of operation. Similarly, superconducting magnets may be particularly important in space technology where light-weight magnets are needed. Such magnets could store more electrical energy in less space than conventional electrical capacitors.

New Superpressure Balloon Developed.—The development and successful testing of a new type of research balloon has been announced by the Air Force Cambridge Research Laboratories. The most significant feature of the development is that the balloon maintains a constant altitude without expenditure of ballast.

Key to the design of the balloon is that its internal pressure exceeds that of the surrounding air. Research on such balloons, called superpressure balloons, has been under way for several years by scientists at AFCRL's Geophysics Research Directorate.

Tests of the newly designed balloon in April were the most successful yet conducted with superpressure balloons. The flight of longest duration took place on April 17. The balloon was spherically shaped with a diameter of 34 ft. Construction was of a Mylar material one and a half mils thick (one and a half thousands of an inch).

The balloon was launched from AFCRL's Vernalis, Calif. test site. It carried a payload of 40 lb. and was designed to float at an altitude of 70,000 ft. Nine days later the test was completed and the flight automatically terminated over the ocean

400 miles off the coast of the State of Washington.

During the nine-day flight, the balloon maintained a constant volume and consequently a constant altitude. This was achieved by inflating the balloon with additional gas to create an overpressure condition at floating altitude. At night the pressure in the balloon decreased due to the loss of radiative heating from the sun, but the volume of the non-extensible balloon remained the same. Conventional zero-pressure balloons decrease in volume after sunset and require the release of ballast to maintain floating during the night. In the case of long duration flights the amount of ballast required can exceed the gross weight of the balloon and its payload.

The flight duration was a record for any type of superpressure balloon. The actual floating altitude was 70,500 ft. Data telemetered from the balloon throughout the flight indicate that the balloon could have continued its flight for a period considerably longer than the nine days.

The results of the test series prove the feasibility of superpressure balloons of this size and payload capacity. A significant step has been achieved toward the development of higher flying, larger payload capacity superpressure balloons that will carry 100 pounds to a constant level altitude of 100,000 ft. for periods of 10 days or more.

Camera Photographs Planets, Satellites in Daylight.—A special Air Force camera that looks like an over-size anti-aircraft gun has been photographing missiles, satellites and planets in broad daylight.

Consisting of 19 long-barrelled 5-in. refracting telescopes linked to as many TV-like image-orthicon tubes, the giant "Facet-Eye Camera" has been installed at the Air Force Missile Development Center, Holloman Air Force Base, New Mexico. The center is one of seven maintained by the Air Force Systems Command.

Moving on a massive but delicately-balanced tracking mount, the system recently captured continuous bright-image views of Venus and Jupiter in broad daylight.

Planetary images—not visible to the naked eye—were transmitted to indoor viewing screens where they were photographed with a precision-plate camera to produce clear, distortion-free pictures ordinarily obtained only during night hours. The total field covered is many times the size obtainable by standard tracking telescopes.

In terms of daylight star detection, this represents an advance of approximately six stellar magnitudes. Scanning the heavens at night, the new camera system has photographed a star cluster, The Pleiades, with image resolution of faint stars up to the 12th magnitude.

The number of telescopes and oscilloscope screens will be increased to 25 to complete the planned design of the Facet-Eye and to provide a viewing field of 2½ by 7 degrees.

Basically conceived for the purpose of tracking missiles, satellites and other space-bound objects at extreme distances and in any kind of light, the system was developed in AFMDC's orbital Mechanics Division by Walter E. Woehl, theoretical and applied physicist. Woehl formerly held high research positions in his native Germany and at the Royal Arsenal in England.

Described as the only instrument of its kind in the world, the system was built to exacting specifications by the International Telephone and Telegraph Laboratories, Fort Wayne, Ind. The 25 telescopes have been furnished by the E. C. Larr Optics Company, Altadena, Calif. The 13,000-lb., four-axis tracking mount was built by Joseph Nunn and Associates, Pasadena, Calif.

New Information System.—Comb-like arms flying on layers of air can monitor as many as 280 million characters of data in an information storage system recently announced by International Business Machines Corporation.

The new 1301 disk storage unit can be linked to any of five of IBM's intermediate to large solid-state computers or shared by any two of them. It transforms them into advanced information centers capable of relaying constantly updated information, by means of IBM Tele-processing devices, to remote points in thousandths of a second.

Each 1301 unit holds from 50 to 56 million characters of information depending on the computer employed. A total of five units can be used with any of the five computers.

"The 1301, with its vast capacity and very high access speeds, opens the way to total systems processing," said Warren C. Hume, president of IBM's Data Processing Division.

"This means the vital records of most businesses can be maintained at a single, convenient location, providing anyone within the company—whether at home office, branch office or plant—almost immediate access to all pertinent data."

"New methods of data organization will enable our major computers to process records in any order without being slowed while files are searched. This significantly extends the range of IBM's in-line random access equipment and makes possible many new processing concepts."

The 1301 is available for use with the IBM 1410, 7070, 7074, 7080 and 7090 computers. The file may be shared by any two of these systems.

Each file contains one or two modules (or stacks) of 20 rapidly revolving disks for data storage. The 1301 has a read/write head for each disk surface. These heads, at the end of comb-like access arms, are aligned in a vertical cylinder. Thus, at any given moment, they are positioned opposite corresponding tracks or grooves on all forty disk surfaces in a module.

This arrangement makes possible the picking up and depositing of data in corresponding data tracks with no access movement. Related information needed for frequent reference can easily be placed in parallel tracks forming concentric cylinders of data for very fast reference. The only motion required within the file is horizontal from data cylinder to data cylinder. There is no necessity for vertical motion by the access arms, as in previous files, because there is a head for every disk surface.

While reading in the same data cylinder, as many as 102,000 characters in a module, 204,000 characters in one disk storage unit, or 1,020,000 characters in a maximum system of five units may be reached with no access motion. These figures vary slightly with the computer used. The access mechanism requires a minimum of fifty milliseconds (thousandths of a second) to move horizontally from one cylinder to the next cylinder within small groupings of tracks on a disk surface. Within larger groupings of fifty tracks, maximum access time is 120 milliseconds. The maximum access motion time to any data in the 1301 is 180 milliseconds. Record organization techniques can cut this time substantially.

The rate at which the 1301 reads characters in sequence into core storage is between 75,000 and 90,000 characters a second, depending on the type of system to which it is linked. This is faster than many tape systems. It is an important factor in sequential jobs such as recalling tables or programs stored on the disks into core memory; or transferring of the file to magnetic tape for a more permanent record of the data.

A flexible record length feature adds to the efficiency of the 1301. Records (units of information) of different lengths may be stored on the cylinders. The capacity of the file, therefore, is much greater than that of fixed length files in which records are placed in identical storage spaces regardless of size.

The read/write heads are part of a gliding shoe designed to fly along the disk surface. These shoes actually glide on a layer of air created by rotation of the disk at 1,800 rpm. There is an equilibrium of forces which keeps the head balanced less than a hair's breadth above the disk surface.

The 1301 makes feasible many large-size or large-volume jobs not previously possible within the computing range of the associated systems.

Such giant tasks as keeping track of the inventory of a large mail order house or the updating of the employment security accounts kept by the state government are within the scope of the 1301 files.

Typically, the 1301 can be used as the basis for an information system for insurance. The detailed policy records, loan accounts and dividend accounts of a life insurance company can be stored in the file. The data would serve the computer for the processing of billing, anniversary calculations and renewals. In addition, it would also be immediately available anywhere in the company for policyholder and agent service.

The loan department, for example, could ask through a remote unit a half-mile away from the central computer the immediate status on policy equities for loan approval on any one of thousands of policies.

As well as extending the jobs a computer can do, the 1301 is capable of increasing the over-all efficiency of present systems.

All of the programs, or job instructions, for an IBM 7090 can be stored on a 1301 unit. Instead of the usual several minutes required to set up a 7090 for a two or three minute processing job, the 1301 can rush another program into the 7090 in seconds, virtually closing time-consuming and costly gaps between jobs. Sharing the same 1301 file, the scientific 7090 and commercial 7080 can have reference to a wide range of both scientific and commercial programs.

Developed and manufactured at IBM facilities at San Jose, California, the 1301 will be shipped to customers starting in the third quarter of 1962.

GaAs Detector Provides Greater Sensitivity.—The Lansdale Division of Philco Corporation has announced a gallium arsenide photovoltaic detector in a T0-18 case which marks the first exploitation of the small signal photo detection capabilities of a solid state device.

Reflecting the growing emphasis on miniaturization and reliability of electronic components used in space systems, Philco has placed the new single crystal photovoltaic detector in the small, rugged T0-18 case, used commonly as a transistor package.

Developed by engineers at the Lansdale Division's Special Products operation, the new device, GAU-401, provides high sensitivity for both visible and near-infrared radiation detection for the first time.

The detector is designed for the 0.4 to 0.9 micron region, having a peak sensitivity at 0.85 microns.

Dr. C. H. Sutcliffe, general manager of the special products operation, explained that the new device provides greater sensitivity than do related photodevices such as photomultiplier tubes and silicon photodiodes.

"This detector," he said, "unlike conventional photo diodes, does not suffer serious sensitivity degradation when exposed to daylight and replaces photo multipliers in providing 1 to 2 orders of magnitude of increased responsivity in the near infrared region."

Dr. Sutcliffe emphasized the device's ability to sense during daylight as being most significant to celestial navigation of satellites and space vehicles.

He explained that due to a high energy gap the GAU-401 can exhibit very high sensitivity at even higher temperatures than those at which silicon devices operate. Operation up to 120° C has shown no loss in detector sensitivity, he said.

He also indicated that the device exhibits virtually no microphonism and can withstand massive radiative overloads without permanent damage.

Dr. Sutcliffe stated that optimum device characteristics are observed at room temperature, obviating the need for dewar-cryostat arrangements associated with conventional IR detectors.

Philco's use of a modified T0-18 case (used for miniature transistors) marks an innovation in detector packaging, providing size reduction and ruggedness.

It is estimated that the GAU-401 detector is 1/20 the size of a related photomultiplier.

Dr. Sutcliffe explained that the miniaturization of the GAU-401 device promises to make possible the combination of a solid state amplifier and photo detector potted in one single block no larger than a matchbox.

The conventional T0-18 case has been modified with a glass window sealed at the top of the cylindrical package which measures 0.18 inch in diameter and 0.20 inch in length (excluding leads).

In the standard configuration, square or circular cell areas ranging from 0.2 to 4.0 mm^2 can be provided.

Dr. Sutcliffe stated that the GAU-401 becomes the first visible-near infrared detector specifically designed for small signal use.

He added that the wide variety of applications which the new detector is suited to include: celestial body sensing, missile tracking, fuzing and warning, space vehicle and satellite attitude control and guidance, stellar supervision of inertial platform, computer readout and numerous industrial process controls.

Sapphires to Protect Phone Satellites.—Thousands of pieces of man-made sapphire will cover the surface of the communications satellites now being developed by Bell Telephone Laboratories, according to the company's president, Dr. James B. Fisk.

The sapphires are expected to protect solar cells from space radiation, enabling "working" telephone satellites to endure the rigors of space for 10 years or more.

Dr. Fisk cited the sapphire protection as an example of thorough-going measures that are being taken to achieve long-life satellite reliability. Bell Laboratories is using an approach it evolved in developing submarine cable systems. Some 1500 electron tubes have functioned under the oceans for the past two to six years without a failure.

"Now the exacting methods that

produced extremely reliable submarine cable systems are being applied to satellites," he said. "The result should be a shipshape, spaceworthy communications relay station."

He pointed out, however, the pressing need for a "real-life test" in space.

Until now, most satellites have been designed for short-term experiments in space. Bell engineers have set as their goal the production of satellites that will work dependably for many years, serving as radio relay stations for communications overseas—telephone calls, data and other forms of communication.

The Bell System has previously announced plans to develop a satellite communication system and put it into operation as soon as possible. A ground station in Maine will be ready early next year. The first test satellite will be ready at the same time if the government designates a launching vehicle soon. The Bell System has offered to pay costs of the rocket and launching.

Dr. Fisk said that the first experimental phone satellite will be roughly spherical, but may have as many as 60 flat surfaces or gem-like facets. The solar cells on each facet would receive an equal amount of light.

The solar cells—invented at Bell Laboratories—are small silicon wafers mounted on the outside surface to convert sunlight to electricity. The sapphire sheathing will protect the solar cells from the deteriorating effects of electron bombardment in space. It will also reduce the effects of proton bombardment.

Two different sizes of test satellite—with diameters of 27 in. and 4 ft.—are being considered for experiments. More than half the surface of the smaller one would be covered by over 4,000 Bell solar cells; the larger satellite, by 12,000 cells.

Covering the cells will be an equal number of slices of man-made, crystal-clear sapphire. Other protective materials are not being ignored, although sapphire appears most promising.

The sapphire does not cost significantly more than other high-quality materials that might be used, and only about one-third of the cost of the solar cells themselves. Sapphire has other advantages. For one, it will convey heat away, preventing the solar cells from being overheated in long periods of continuous sunlight.

Another advantage is that the use of sapphire overcomes the effects of very rapid and extreme changes of temperature in space. Most materials expand when heated, but by different amounts. If the rate of expansion differs too much, the materials will not stay fastened together.

The sapphire slices will be brazed to platinum sidebars which will be soldered to a ceramic base. These materials were selected because they all expand at very nearly the same rate.

In addition, sapphire, being hard, will resist erosion by micrometeorites.

Under the reliability approach evolved in the development of submarine cable and other systems, the goal is to assure a predictable long life for the entire system. To do this, engineers must develop each individual component to such a reliability that its individual predicted life-length is far greater than the goal for the over-all system.

"First they must study the environment—whether it is two miles beneath the sea or 4,000 miles above it—and define all the conditions and hazards," Dr. Fisk said.

Then, in an artificially simulated environment, they intensify the effects of the hazards to make the components fail. The engineers then redesign until they can predict accurately the reliability of a required component. Finally, they conduct a real-life test program to confirm that the system performs as predicted.

Unfortunately, the space environment is not yet known precisely. The exact amounts of radiation in space at various altitudes and under varying circumstances have not been measured. Therefore Bell Laboratories engineers plan to use their first experimental satellite both to take additional measurements of space radiation and to serve as a first "real-life test" of components planned for a working satellite system.

Heat-Shrinkable Tubing.—The development of a radically new type of tubing that shrinks to form a tight mechanical bond within 7 seconds following application of heat (135° C) has been announced by Alpha Wire Corp., New York, N. Y.

Marketed under the name of Alphlex Shrinkable Tubing, the new tubing is an irradiated, flame-retardant, thermally stable, modified polyolefin base product, with applications limited only to the imagination of the design engineer. It is manufactured in an expanded form so that it may be easily slipped over wire and cables, hosing, terminals or conduit. When heat of approximately 135 C. is applied, the material shrinks to a reduced, predetermined diameter forming a secure, tight-fitting bond. Longitudinal shrinkage is less than 10 per cent. To supply the heat the use of a heat gun is recommended. However, satisfactory results may be obtained by oven heating, radiant heat, dipping in hot liquid, ovens, soldering iron or open flame.

Alphlex Shrinkable Tubing is marketed in standard packages of 4-ft. lengths. Sizes 24 through 4 are available in black, white, red or yellow; sizes 2 through 1 (in.) available in black; other colors and lengths may be obtained on special order.

When used as an insulating material, Alphlex Shrinkable Tubing has excellent temperature characteristics and remains flexible and strong from −55 C. to 135 C. continuous. Within this temperature range it will not melt, harden, run, crack, or blister. The high flexibility of Alphlex Shrinkable Tubing remains constant throughout the expansion and reduction process. Adding to its application as an insulating material is the high dielectric and mechanical strength of Alphlex Shrinkable Tubing and its resistance to fungus growth and moisture absorption. When used in hydraulic processes, Alphlex Shrinkable Tubing provides a flexible, chemical-resistant jacket or coupling.

Other applications for Alphlex Shrinkable Tubing include use as insulation for cables, harnesses, pigtails, bus bars and electronic components. Alphlex Shrinkable Tubing also may be used to protect terminals, to mend or join shafts, as a convenient insulation for tool handles, and for permanent, easy to use, cable markers and color coding. Alphlex Shrinkable Tubing is easily marked or stamped with hot stamping equipment.

Alphlex Shrinkable Tubing has an exceptionally long shelf-life and will retain its expanded diameter until heated. No radioactivity remains in or on the material after irradiation.

Private Satellite Telephone.—You could have a private telephone "line" between your own location and your business headquarters or friends' homes many thousands of miles away with the space communication satellite system proposed by the General Electric Company. And the contact could be made by a simple dialing of a telephone as you do now in the home or office.

This feature of the proposed system was explained by A. G. Steinmayer, Jr., systems project engineer at the Company's Missile and Space Vehicle Department in Phildelphia, in a paper presented at the National Symposium on Global Communication there.

Mr. Steinmayer said an isolated business office in Australia could lease the service on a monthly basis and have a constant contact with London or with another office 75 miles away if desired. The satellite system would provide high quality reception at both terminals because the signals would not be affected by cloud cover or adverse weather conditions.

A person living in an isolated area wishing to use the system would be equipped with broadcasting and receiving equipment that would "tune in" on the radio frequencies assigned to him. By picking up his telephone, and dialing, he would be in contact with the satellite in his area. His signals would then be relayed by the satellite to a neighbor's ground station or to a commercial ground station which could handle his call through an adjoining ground communication system.

The proposed General Electric system would employ ten or more satellites to cover the earth's populated areas. Each satellite would have 600 two-way voice channels. As one satellite passed out of range, another satellite would take over. Signals would go from a ground station to the satellite and down to a ground station. If the call were placed to the other side of the earth, the signals would go up to a satellite and back to a ground station several times. This enables the same channels to be used in at least nine different areas of the earth and makes it possible for thousands of users to enjoy the benefits of satellite communication service.

A major advantage of the General Electric system is that is could be used by all—small as well as large countries and private individuals. The cost for an isolated private individual would be high compared to an average urban phone bill but quite moderate, in terms of the service provided, for an isolated user.

As proposed by General Electric, its system is designed to benefit the entire populated world, rather than just the heavily populated and developed areas.

General Electric recently organized a new company—Communication Satellites, Inc.—to operate a space satellite communication system and has applied to the Federal Communications Commission for an operating license. This organization would lease either individual channels or blocks of channels to users.

The lease arrangement vastly simplifies the monitoring and billing problem and completely obviates the need for the normal type of switchboard operation.

In operation, each user would have a range of frequencies assigned to him that would carry the radio signals. Messages would be carried simultaneously as they are on telephone wires today. This is technically known as frequency stacking messages.

The system that is currently considered to provide optimum benefits employs pulse code modulation techniques. This is transmission of the voice signals in pulses that are decoded at the receiving end. Full tonal fidelity is preserved so that the user's voice loses no personality or inflection characteristics.

Machine Detects One Odd Atom in Crowd of a Billion.—A 4-ton research machine that detects one foreign atom lurking among more than a billion atoms—a feat comparable to picking out one wanted man hiding in a crowd of all the people in North and South America, Europe, and Africa—has been put to work at RCA's David Sarnoff Research Center in Princeton, N. J.

The new instrument, known as a spark-source mass spectrograph, is one of the world's most sensitive research tools. It is being used by RCA scientists to probe new electronic materials that will be used in tomorrow's computers, communications, and control systems. Built for RCA Laboratories by Associated Electrical Industries (Manchester), Ltd., in Manchester, England, the machine was delivered by transatlantic air freight for installation at the Princeton center.

Announcing the start of research operations with the new instrument, H. W. Leverenz, Director of Research, RCA Laboratories, said:

"Basic research into the nature and properties of materials that have potentially useful electronic characteristics has become a principal foundation for the advances that we anticipate for the future in electronics. RCA's extensive and expanding program in materials research already has produced many results of both fundamental and practical importance. These materials make possible new or improved devices now entering use in computers, television systems, microwave communications, satellites, and a multitude of defense electronic systems. This new precision instrument, added to the advanced facilities now in use at RCA Laboratories, will enable our scientists to probe even more deeply into the composition of promising new materials, and thus to accelerate the development of new building blocks for tomorrow's electronic devices and equipment."

Mr. Leverenz described the new mass spectrograph as an instrument whose sensitivity is "unmatched by that of any such equipment known to be in use in this country." Describing its ability to pick out and identify in only an hour, one impurity atom from among more than a billion atoms, he pointed out that even this minute degree of variation from a desired composition can affect the electronic properties of a material.

The operating principle of the mass spectrograph is based on the fact that each chemical element has a distinctive mass and ionization characteristic distinguishing it from all other elements. In the spark-source mass spectrograph, a small amount of the material to be analyzed is converted by an electric spark discharge into a cloud of positively-charged atoms, or positive ions. The charged atoms are accelerated, then sent in a narrow beam through electrical and magnetic fields which deflect them in varying degrees, depending upon their velocity and mass. The atoms of different elements are thus separated, to make impact at different points along a precisely calibrated scale on a photographic plate. The result is a detailed record of all the elements present even in extremely minute quantities in the material that is being analyzed.

Publications Available from Library of Congress.—The Library of Congress has a duplicate stock of back numbers of its publications, the *East European Accessions Index* and the *Monthly Index of Russian Accessions*, available for donation to libraries of educational institutions and public bodies in the United States. Serving as keys to new literature coming into this country from the USSR and East Europe, these publications are of value to scientists and other scholars. They are more than mere "booklets," for they contain information about the content of new Russian and East European books and articles. Each index gives in English a monthly account of new material in a variety of subject fields as received from the USSR and East Europe by the Library of Congress and by other American research libraries. The translation of all books and articles into English and the elaborate subject guides in English permit a researcher who has no command of Russian or East European languages to identify easily the material important to him.

The number of copies available of individual issues varies for both of these monthly publications, but the Library will undertake to fill requests for nearly complete back files insofar as the supply permits. All requests, whether for single issues or for all numbers available, will be filled in order of receipt. The requests should be addressed to the Chief, Exchange and Gift Division, Library of Congress, Washington 25, D. C.

Subscriptions for future issues of these publications may be placed with the Superintendent of Documents, United States Government Printing Office, Washington 25, D. C.

"Plug Nozzle" Rocket Engine.—A new type rocket engine whose design promises substantial cost and time savings in narrowing the gap between American and Soviet rocket propulsion systems has been test fired by General Electric.

Called the "plug nozzle" engine and termed the "first radical change in rocket engine design in more than a decade," the powerplant developed more than 50,000 lb. of thrust in a recent firing at General Electric's Rocket Test Station in Malta, N. Y.

The company's propulsion engineers claim the engine can be scaled up to multi-million-pound thrust levels in less time and at less cost than today's conventional rocket propulsion systems.

The feasibility of the G-E pioneered plug nozzle rocket engine concept, under study for several years at the firm's Flight Propulsion Laboratory Department, is being demonstrated under contract with the National Aeronautics and Space Administration. Recent firings of the engine are part of an investigation being conducted by the space agency.

Dr. Kurt Berman, head of G.E.'s Rocket Test Station, said that the plug nozzle firing represents a major advance in the development of liquid propellant rocket engines of unconventional design. "The plug nozzle," he said, "is the first radical change in rocket engine design in more than a decade. We believe this engine can materially increase reliability and reduce the time and cost of developing high-thrust rocket engines."

The plug nozzle engine differs from the conventional rocket engine both in appearance and development approach. While existing rocket engines have an exhaust chamber which is bell-shaped, the plug nozzle is only half as long and is shaped like a funnel.

According to G. E. engineers, increasing the thrust of conventional-type engines requires that the propellant injector be enlarged. However, this creates critical combustion problems which can only be overcome with a major engineering and development effort. The results are costly, both in terms of long development cycles, and expenditures for constantly changing hardware and high-thrust test facilities.

The plug nozzle approach to higher thrust levels eliminates the need for "cut and try" methods of development. Combustion in the new engine occurs in small segmented combustor cells located around the periphery of the conical plug. These small cells effectively reduce the combustion problem, and increase engine reliability. By increasing or decreasing the number of these cells, engines of various thrust levels can be built without developing a new injector for each engine size.

"We can test segments of these cells," said Berman, "and then group the segments together to get as much thrust as we want. We don't have to take the intermediate steps."

The plug nozzle is being studied for use as a multi-million-pound booster for large payloads, as an upper-stage engine and for space station propulsion. It has a decided advantage over conventional engines in that varying the pressures in the combustor segments affords a simple means to "steer" the rocket.

Conventional engines, on the other hand, require a bulky gimballing structure to do the same thing. The gimballing structure mechanically turns the conventional engine to change the direction of thrust in respect to the direction of the rocket.

United Kingdom's VERA Reactor.—The major purpose of the new VERA reactor at the Atomic Weapons Research Establishment, Aldermaston, England, which went critical for the first time late in February, is to gather information in an area where it has up to now been substantially lacking, that is, that of medium-enrichment, of diluted and of slightly moderated fast reactor systems.

The reactor cell has walls 6 ft. thick and a ceiling 3 ft. thick. Around the cell are located the control room, the fuel element assembly room, laboratories, plant room for the ventilation system and filters and a laboratory to house a 600-kV accelerator, still under construction. This accelerator will be used to produce deuteron beams aimed at targets in sub-critical VERA cores which will yield bursts of neutrons in a series of kinetic experiments to measure reactivity increments and neutron lifetimes and to study neutron spectra with varying core compositions.

Versatility is contributed to the reactor, and its name, (VERA—Versatile Experimental Reactor Assembly) by the fuel element, which could hardly be more simple. Each element consists of two stainless steel U-channels into which $\frac{1}{8}$-in. thick, roughly square "biscuits" of enriched uranium, natural uranium and graphite are stacked in numbers and patterns which can be selected to correspond to practically any fuel dilution or degree of moderation.

Only the central portion of each element is occupied by this mixture, the remainder of the stack consisting of thicker blocks of natural uranium or other materials serving as the reflector.

A small gap is left between the two channels when they are clamped together so that the fuel element make-up can be checked at a glance—the enriched (93%) biscuits are coated with orange lacquer and the natural ones with green. This lacquer also helps to reduce the spread of radioactive dust.

The reactor is brought into operation by raising eight fuel elements, so that their central enriched portion enters the core. They are suspended by steel cables from winding drums equipped with electromagnetic clutches. On quick release, all control and safety elements fall, out of the core, under gravity, closing the device down immediately.

Closed circuit TV is used to watch operations from the control room.

Computer to Serve Advanced Technologies.—A new scientific computer, designed to meet the advanced data processing needs of complex technologies, has been announced by International Business Machines Corporation.

The IBM 7072 will be able to process magnetic tapes recorded during engineering tests—jet aircraft wind tunnel trials, for example—despite a difference in the "languages" of the test cell and the computer. Binary data are entered directly into the computer, where they are converted into decimal form automatically and processed for immediate use by engineers.

The new system is suited especially for problems with an emphasis on powerful computing ability, rather than high-volume input and output of data.

The latest member of a fully-compatible family of computers, which includes the IBM 7070 and 7074 data-processing systems, the 7072 is expected to serve a wide range of users, including aircraft manufacturers, engineering organizations and heavy industry.

"At a time when science and industry are placing increased reliance on computing as they push into new fields, the 7072 offers great potential for intermediate-cost, high-powered technical data processing," Y. P. Dawkins, director of marketing for IBM's Data Processing Division, said. "The new system also provides broad business data processing capabilities."

Among the scientific and engineering problems that will be handled by the 7072 are: the creation of mathematical models to test product designs, systems and operational theories; simulation of space flight vehicle trajectories; design of nuclear reactors and supersonic aircraft; analysis of products and structures to determine their ability to withstand stress; reduction of large volumes of data to a form in which they can readily be analyzed; and solution of equations using such general mathematical techniques as matrix algebra and vector analysis.

The 7072, with more than two times the internal speed of the IBM 704, has access to data stored in core storage memory in six-millionths of a second.

The availability of 7070/7074 automatic programming systems will permit users to test their programs at any IBM Datacenter prior to the installation of the 7072 and to put the system into immediate operation after it has been installed. These programs include IBM's FORTRAN, AUTOCODER, SORT and MERGE, REPORT GENERATOR and IOCS (Input Output Control System).

Device Analyzes Binary Gas Mixtures.—The Gasalyzer, a new battery-operated portable instrument which accurately measures flue and auto exhaust gases, is now being marketed by the Industrial Equipment Division of Engelhard Industries, Inc., Newark, N. J.

The device, which weighs only 6½ lb., is capable of analyzing binary gas mixtures to within ± 5 percent accuracy. Higher degrees of accuracy can be obtained with special calibration.

Developed originally for use by the U. S. Army Quartermaster Corps, the Gasalyzer readily checks combustion efficiency and can be adapted to measure hydrogen, sulfur dioxide and carbon dioxide. It also can be used to measure fuel-air ratios in internal combustion engines, turbines and furnaces.

Powered by a standard 6-volt, dry cell battery, the Gasalyzer will operate as long as its battery has life. A dying battery will not affect performance, and only when the electrical source is dead will the instrument stop functioning. During testing, batteries lasted for 72 continuous hours.

The Gasalyzer can be adapted to 110 volts a.c. by using a power pack available at additional cost.

The operation of the Gasalyzer is based on the thermal conductivity principle using two sensory elements in a wheatstone bridge. The gas passes through an analysis cell, causing the filament of the cell to heat or cool with respect to the filament of a reference cell. The temperature change of the analysis cell results in a resistance change which upsets the bridge balance. This change then is reflected on a highly sensitive meter.

Photographic Exposure Repeater for Transistor Industry.—The first commercially available photographic exposure repeater able to produce extremely precise photo-masks required by the transistor industry has been developed by the David W. Mann Company, Lincoln, Mass., a division of the Geophysics Corporation of America.

Built around a two-coordinate comparator of the type which has identified the company as a world leader in the very exact science of metrology, the new equipment permits positioning of patterns to an accuracy of one micron (.0004 in.), exact focusing, and a precise ratio of reductions. The Repeater has a programming mechanism capable of performing various combinations of step-and-repeat operations for automatic exposures, although the instrument may also be operated manually for specially spaced exposures.

A major feature of the new Repeater is a three-pin plate positioning mount on the plateholder which permits accurate positioning and exposure of an aligned series of plates. A mask use fixture of identical configuration makes it possible to insert a series of photo-masks and produce precisely aligned successive exposures of semiconductor material.

The repeater is available in three basic models, an Industrial, Research, or Manual model. The instrument's major advantages lie in cost, time savings, and proprietary security in the production of masks, and convenience in experimentation with new types of masks.